

Unit 1 Notes / Secondary 3 Honors

Day 1: FUNCTIONS

FUNCTION: a relation that assigns to each element (x) exactly one element (y)

- each x (input value) is used once.

DOMAIN: Set of all x-values
(input)

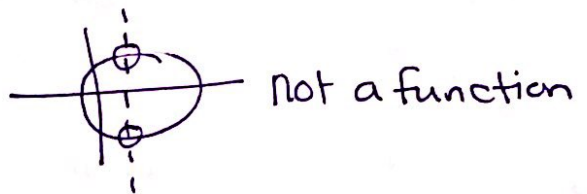
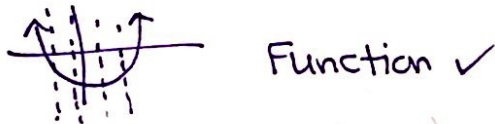
RANGE: Set of all y-values
(output)

TESTING FOR FUNCTIONS:

We can determine if a relation is a function by looking at *graphs, lists of coordinates, and equations.*

* To determine if a relation is a function by looking at its GRAPH:

* Vertical line Test



* To determine if a relation is a function by looking at a list of its COORDINATES:

* are all the x's different?

(1, 2) (3, 4) (5, 6) (6, 4)

yes

(1, 2) (1, 3) (1, 6)

NO

* To determine if a relation is a function by looking at its EQUATION:

* Do I get more than one value for y when I plug in x's?

$$x + 2 = y^2$$

$$\pm \sqrt{x+2} = y \rightarrow \text{if plug in } x=2, y=2$$

$$y = -2 \Rightarrow \text{if } y^2 \text{ then not a function}$$

Not a function

Examples: Determine whether the relation represents y as a function of x.

1) $\{(2,11), (2,10), (3,8), (4,5), (5,1)\}$

No

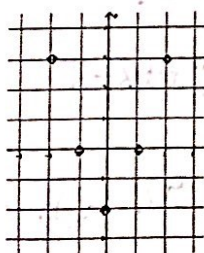
2) $\{(a,2), (b,3), (c,4)\}$

Yes

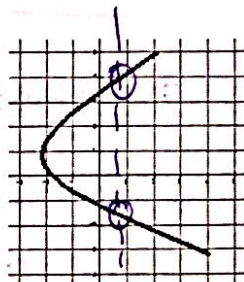
3) $\{(4,2), (3,2), (1,2)\}$

Yes

4)



5)



Which of the equations represent(s) y as a function of x ? Explain.

6) $x^2 + y = 1$

Yes

7) $-x + y^2 = 1$

NO

8) $x^2 + y^2 = 1$

NO

FUNCTION NOTATION

For equations where y is written in terms of x ; $y = (\text{some equation})$ are often given names such as f or g if they are functions. For example $y = 3x^2 - 2$ could be written as $f(x) = 3x^2 - 2$. This is known as function notation where $f(x) = y$. Function notation can also be used to indicate values for x : $f(3)$ means the value of f when $x = 3$.

Examples: Evaluate $g(x) = -x^2 + 4x + 1$ at the following points:

9) $g(2)$
 $= -(2)^2 + 4(2) + 1$
 $= -4 + 8 + 1$
 $= \boxed{5}$

10) $g(m)$
 $= \boxed{-m^2 + 4m + 1}$

11) $g(x+2)$
 $= -(x+2)^2 + 4(x+2) + 1$
 $= -(x^2 + 4x + 4) + 4x + 8 + 1$
 $= -x^2 - 4x - 4 + 4x + 8 + 1$
 $= \boxed{-x^2 + 5}$

12) Find all real values of x such that $f(x) = 0$: $f(x) = 5x + 1$

$0 = 5x + 1$

$x = \boxed{-1/5}$

DIFFERENCE QUOTIENT

The formula for the difference quotient is $\frac{f(x+h) - f(x)}{h}$. It is related to the slope equation.

EX) Find the difference quotient and simplify the answer:

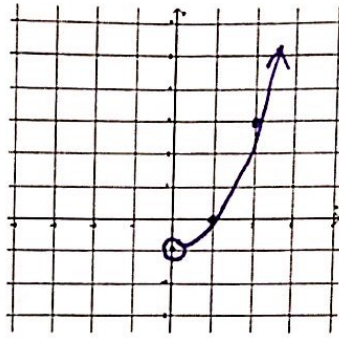
13) $f(x) = 2x - 1$
 $= \frac{2(x+h) - 1 - (2x - 1)}{h}$
 $= \frac{2x + 2h - 1 - 2x + 1}{h}$
 $= \frac{2h}{h} = \boxed{2}$

14) $g(x) = x^2 - 4x + 7$
 $= \frac{(x+h)^2 - 4(x+h) + 7 - (x^2 - 4x + 7)}{h}$
 $= \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}$
 $= \frac{h^2 + 2xh - 4h}{h} = \frac{h(h + 2x - 4)}{h} = \boxed{h + 2x - 4}$

DOMAIN RESTRICTIONS: Sometimes the domain of a function will be included with the equation:

EX) $f(x) = x^2 - 1, x > 0$ Sketch this graph

Domain: $x > 0$



→ Values of x you can't use in the function

Two reasons for domain restrictions are:

→ Fractions:

Can't have 0 in denominator

EX) Find the domain of each example:

16) $f(x) = \sqrt{x}$

$x \geq 0$

17) $h(x) = \frac{1}{x+5}$

all $\mathbb{R}, x \neq -5$

18) $g(x) = \frac{1}{x^2-4}$

all $\mathbb{R}, x \neq 2, -2$

$x^2 - 4 = 0$
 $x^2 = 4$
 $x = \pm 2$

→ Even radicals: $\sqrt{\quad}, \sqrt[4]{\quad}, \sqrt[6]{\quad}, \dots$

Can't have a negative value under even radicals.

19) $h(x) = \sqrt{x^2 - 4}$
 $x^2 - 4 \geq 0$



$x \leq -2$
 $x \geq +2$

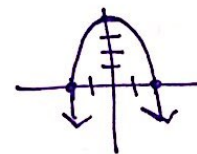
20) $p(x) = \sqrt{x+3}$

$x+3 \geq 0$

$x \geq -3$

21) $k(x) = \sqrt{4-x^2}$

$4-x^2 \geq 0$



$-2 \leq x \leq 2$