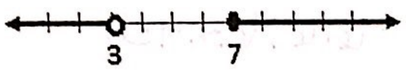
Day 4: **GRAPHS of FUNCTIONS**

Domain: *The set of all x values for which a function is defined.*
Range: *The set of all y values given by the function.*
Absolute Maximum: *The largest **y value** obtained by a function.*
Absolute Minimum: *The smallest **y value** obtained by a function.*

* When writing the domain and/or range: Included endpoints: $\geq$ $\leq$ OR [ ]

Excluded endpoints: $>$ $<$ OR ( )

Examples: Rewrite the inequalities using BOTH Inequality Notation and Interval Notation:

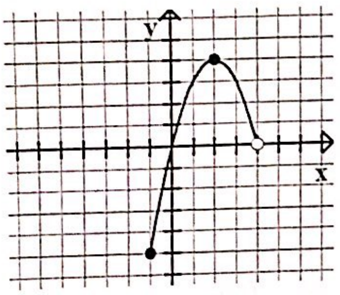
1) 

Inequality: $x<3$ $x\geq 7$
Interval: $(-\infty,3)$ $[7,\infty)$

2). 

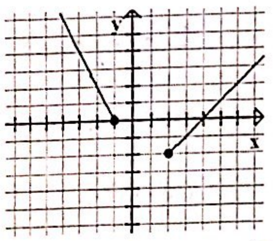
$-2\leq x<4$
$[-2,4)$

A graph of a function can be used to find its domain, range, and various function values.



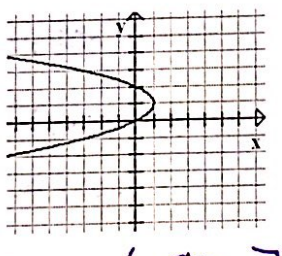
Domain: $[-1,4)$
Range: $[-5,4]$
Abs Max: 4
Abs Min: -5

f(2) = 4
f(3) = 3
f(-1) = -5
f(4) = undefined



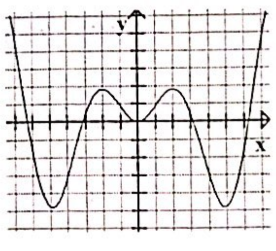
Domain: $(-\infty,-1]$ $[2,\infty)$

Abs Min: -2



Domain: $(-\infty,1]$

Function: yes/no



Domain: $\mathbb{R}$

Abs Max: none

***RELATIVE MINUMUM and MAXIMUM VALUES** → highs & lows

Using your calculator, find the points at which the relative minimum and relative maximum values occur for the following. (Use the calc. menu – 2nd trace). You may have to adjust your windows!

3) $f(x)=3x^2-4x-2$

Relative Min: (.67 , -3.$\bar{3}$)

Relative Max: none

4) $g(x)=-x^3+x$

Relative Min: (-.58 , -.38)

Relative Max: (.58 , .38)

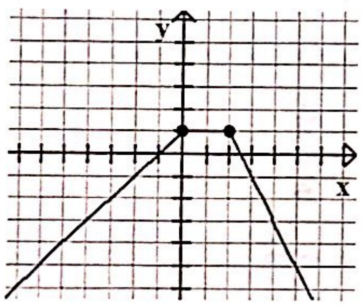
5) $y=0.026x^3-1.03x^2+10.2x+34$

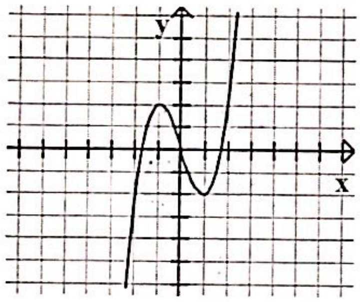
Relative Min: (19.81 , 33.98)

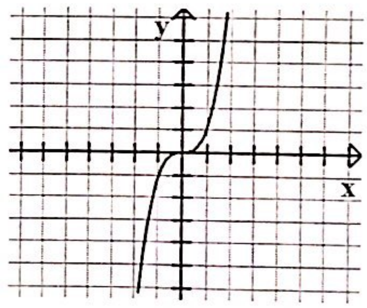
Relative Max: (6.6 , 63.93)

*** INCREASING AND DECREASING FUNCTIONS (x-value intervals) (x_1,x_2)**

Determine the intervals on which each of the following functions is increasing, decreasing, and constant. The intervals should be listed as *open intervals* using *x values*!







6) $f(t)=\begin{cases} t+1, & t<0 \\ 1, & 0\le t\le 2 \\ -2t+5, & t>2 \end{cases}$

Increasing: $(-\infty, 0)$

Decreasing: $(2, \infty)$

Constant: $(0, 2)$

7) $g(x)=x^3-3x$

Increasing: $(-\infty, -1)$ $(1, \infty)$

Decreasing: $(-1, 1)$

Constant: none

8) $f(x)=x^3$

Increasing: $(-\infty, \infty)$

Decreasing: none

Constant: none

9) In problem #4 above, give the intervals where the graph is increasing and decreasing.

Dec. $(-\infty, -.58)$ $(.58, \infty)$ Inc. $(-.58, .58)$

*** INTERCEPTS and SOLUTIONS**

The point (a , 0) is an x-intercept of the graph of an equation if it is a solution point of the equation. To find x-intercepts, let y = 0 and solve for x.

The point (0 , b) is a y-intercept of the graph on an equation if it is a solution point of the equation. To find y-intercepts, let x = 0 and solve for y.

Examples: Without graphing, find the x and y intercepts of the following:

10) $2x+3y=5$

$X: 2x+3(0)=5$

$2x=5, \quad x=5/2$

$Y: 2(0)+3y=5 \quad y=5/3$

x-intercept: $(5/2, 0)$

y-intercept: $(0, 5/3)$

11) $x^2y-x^2+4y=0$

$X: x^2(0)-x^2+4(0)=0$

$-x^2=0$

$x=0$

$Y: (0^2)y-0^2+4y=0$

$4y=0$

$y=0$

x-intercept: $(0,0)$

y-intercept: $(0,0)$

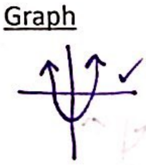
*** SYMMETRY**

A graph has symmetry with respect to the **y-axis** if, whenever (x , y) is on the graph, so is (-x , y).

12) $y=x^2-2$ Test: Graph

$y=(-x)^2-2$

$y=x^2-2$



Equation is the same so

yes y axis sym.

13) $y=(x-2)^2$ Test: Graph

$y=(-x-2)^2$

* NOT the same

so

NO

A graph has symmetry with respect to the **origin** if, whenever (x , y) is on the graph, so is (-x , -y).

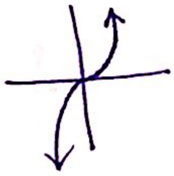
14) $y=x^3$ Test: Graph

$-y=(-x)^3$

$-y=-x^3$

$y=x^3$

* same equation so yes

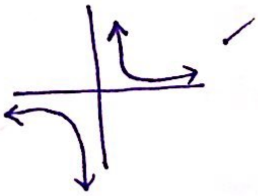


15) $y=\frac{1}{x}$ Test: Graph

$-y=\frac{1}{-x}$

$y=\frac{1}{x}$

yes



**A graph which is symmetric to both the x-axis and the y-axis is then also symmetric to the origin.

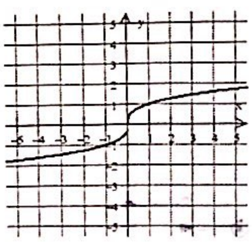
??? Will a function ever be symmetric to the x-axis? Why or why not?????

NO

Will not pass the vertical line test.

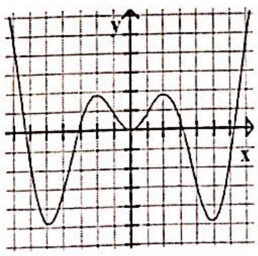
EXAMPLES of TYPES OF SYMMETRY:

18)



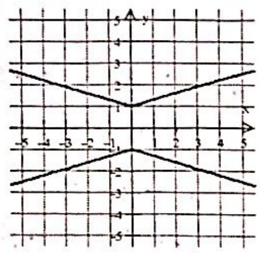
Origin

19)



y-axis

20)



X-axis

*** EVEN AND ODD FUNCTIONS**

A function whose graph is symmetric with respect to the **y-axis** is an **EVEN** function.
A function is even if, for each x in the domain of f, $f(-x) = f(x)$. (same as saying $(x,y) \to (-x,y)$)
To test if a function is EVEN:

* plug in $(-x)$ for x and simplify - If equation stays the same it is even. (same as y-axis symmetry)

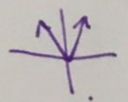
A function whose graph is symmetric with respect to the **origin** is an **ODD** function.
A function is odd if, for each x in the domain of f, $f(-x) = -f(x)$.
To test if a function is ODD:

* Plug in $(-x)$ for x and $(-y)$ for y and simplify. If equation stays the it is odd. (same as origin symmetry).

Examples: Determine if the function is *even, odd, or neither.* Look at it graphically, algebraically, or informally:

21) $f(x) = |x|$

$\boxed{\text{EVEN}}$



y axis

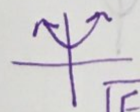
$y = |-x|$
$y = |x|$ ✓

22) $f(x) = x^3 - x$

even? $y = (-x)^3 - (-x)$
$= -x^3 + x$ no

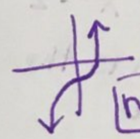
ODD? $-y = (-x)^3 - (-x)$
$-y = -x^3 + x$
$y = x^3 - x$ $\boxed{\text{ODD}}$

23) $f(x) = x^2 + 1$



$\boxed{\text{EVEN}}$

24) $f(x) = x^3 - 1$



$\boxed{\text{neither}}$

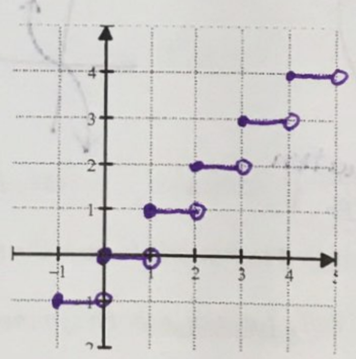
*** STEP FUNCTIONS**

Step functions have "jumps" in their graphs which create the appearance of steps. They are called step functions. The most common example is the greatest integer function. It is denoted by $f(x) = [x]$ or $[\![x]\!]$.
On the calculator it is denoted by f(x) = int x (math → numbers).

$[x]$ or int. x finds the greatest integer less than or equal to x.

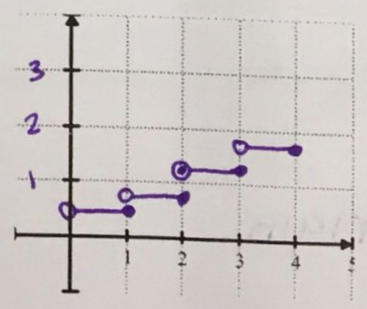
Graph f (x) = $[x]$ on your calculator (dot mode).

* make sure to use open & closed dots correctly.



25) Suppose the cost of a telephone call between Los Angeles and San Francisco is \$0.50 for the first minutes and \$0.36 for each additional minute. Write a model (equation) for the cost of this call. Sketch the graph of the function.

$$f(x) = \begin{cases} .5 & 0 < x \le 1 \\ .86 & 1 < x \le 2 \\ 1.22 & 2 < x \le 3 \\ 1.58 & 3 < x \le 4 \\ \vdots & \vdots \end{cases}$$



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1-5: COMBINATION of FUNCTIONS

☺ OBJECTIVES: Add, subtract, divide, and multiply functions. Compute function compositions.

ARITHMETIC COMBINATIONS of FUNCTIONS

Two functions can be combined to create new functions by using the following properties:

SUM: $(f+g)(x) =$ $f(x) + g(x)$

DIFFERENCE: $(f-g)(x) =$ $f(x) - g(x)$

PRODUCT: $(fg)(x) =$ $f(x) \cdot g(x)$

QUOTIENT: $\left(\dfrac{f}{g}\right)(x) =$ $\dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

Examples: Use the functions $f(x) = 2x+1$, $g(x) = x^2+2x-1$, & $h(x) = 2$ to find the following:

1) $(f+g)(x)$

$= (2x+1) + (x^2+2x-1)$

$= \boxed{x^2+4x}$

2) $(h-f)(x)$

$= 2 - (2x+1)$

$= 2 - 2x - 1$

$= \boxed{-2x+1}$

3) $(gh)(x)$

$= (x^2+2x-1)(2)$

$= \boxed{2x^2+4x-2}$

4) $\left(\dfrac{h}{f}\right)(x)$

$= \boxed{\dfrac{2}{2x+1}}$

For the functions $f(x) = x^2 - 1$ and $g(x) = x - 2$, evaluate the following:

5) $(f-g)(-2)$

$= ((-2)^2 - 1) - (-2-2)$

$= 4 - 1 + 4$

$= \boxed{7}$

6) $(fg)(4)$

$= ((4)^2 - 1)(4-2)$

$= (16-1)(2)$

$= (15)(2) = \boxed{30}$

7) $(f+g)(4m)$

$= ((4m)^2 - 1) + (4m - 2)$

$= 16m^2 - 1 + 4m - 2$

$= \boxed{16m^2 + 4m - 3}$

8) $\left(\frac{f}{g}\right)(0)$

$= \dfrac{f(0)}{g(0)} = \dfrac{-1}{-2} = \boxed{1/2}$

Domain: All x's in domain of g(x) minus any x's that give y-values that don't carry through to f(x).

COMPOSITION of FUNCTIONS f(g(x))

A composition of functions is a function chain. The range of one function becomes the domain for another.

For example, the composition of the function f with the function g is denoted by $(f\circ g)(x)=f(g(x))$. We say this as "f composite g." In general, $(f\circ g)(x)\neq(g\circ f)(x)$.

f of g of x

Example: Let $f(x)=x^2-1$, $g(x)=\sqrt{x+1}$, and $h(x)=2$. Find each of the following and their domains where applicable.

g (D: $x\ge -1$, R: $y\ge 0$) → f D: all ℝ

f D: all ℝ, R: $y\ge -1$ → g D: $x\ge -1$

9) $(f\circ g)(x)$

$=f(g(x))=f(\sqrt{x+1})$

$=(\sqrt{x+1})^2-1$

$=x+1-1$

$=\boxed{x}$

D: $x\ge -1$

10) $(g\circ f)(x)$

$=g(f(x))=g(x^2-1)$

$=\sqrt{x^2-1+1}$

$=\sqrt{x^2}$

$=\boxed{x}$

D: all ℝ

11) $(g\circ f)(-4)$

$g(f(-4))=g((-4)^2-1)$

$=g(15)$

$=\sqrt{15+1}$

$=\sqrt{16}=\boxed{4}$

12) $(f\circ f)(x)$

$=f(f(x))=f(x^2-1)$

$=(x^2-1)^2-1$

$=x^4-2x^2+1-1$

$=\boxed{x^4-2x^2}$

D: all ℝ

13) $(g\circ g)(3)$

$=g(g(3))=g(2)$

$=\sqrt{2+1}$

$=\boxed{\sqrt{3}}$

14) $(h\circ g)(x)$

$h(g(x))=h(\sqrt{x+1})$

$=\boxed{2}$

EX) Determine the domains of $f, g,$ *and* $f\circ g$:

15) $f(x)=\dfrac{1}{x}$ $g(x)=x+7$

g	f	f∘g
D: ℝ	D: ℝ, $x\neq 0$	ℝ, $x\neq -7$
R: ℝ		

$0=x+7$

$-7=x$

16) $f(x)=\dfrac{1}{x^2-4}$ $g(x)=x+2$

g	f	f∘g
D: ℝ	D: ℝ, $x\neq 2,-2$	ℝ, $x\neq 0,-4$
R: ℝ		

$x+2=2$, $x=0$ $x+2=-2$, $x=-4$

DECOMPOSING A FUNCTION

Find $f(x)$ and $g(x)$ if $h(x)=(f\circ g)(x)$ means to "undo" a composite function. It helps to look for an "inner" and "outer" function.

Examples: Given $h(x)=\dfrac{1}{(x-2)^2}$, for which of the following pairs of functions is $h(x)=f(g(x))$?

17) $g(x)=\dfrac{1}{x-2}$, $f(x)=x^2$

$f(g(x))=\left(\dfrac{1}{x-2}\right)^2=\dfrac{1}{(x-2)^2}$

~~18)~~ $g(x)=x^2$, $f(x)=\dfrac{1}{x-2}$

$f(g(x))=\dfrac{1}{x^2-2}$

~~19)~~ $g(x)=\dfrac{1}{x}$, $f(x)=(x-2)^2$

$f(g(x))=\left(\dfrac{1}{x}-2\right)^2$

20) $g(x)=x-2$, $f(x)=\dfrac{1}{x^2}$

$f(g(x))=\dfrac{1}{(x-2)^2}$

Examples: Find two functions f and g such that $(f \circ g)(x) = h(x)$:

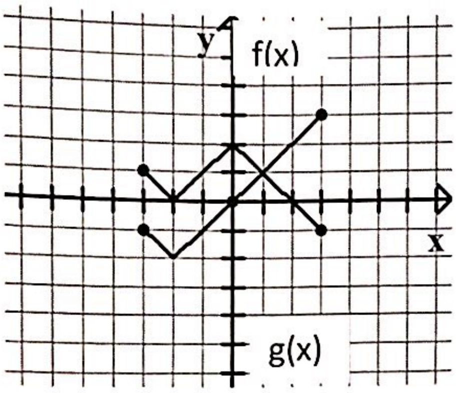
21) $h(x) = (1-x)^3$

$f(x) = x^3 \quad g(x) = 1-x$

22) $h(x) = \sqrt{9-x}$

$f(x) = \sqrt{\cancel{x}} \quad g(x) = 9-x$

Given this graph, find:



23) $f(-2) = \boxed{0}$

24) $g(2) = \boxed{2}$

25) $(f+g)(-1)$

$f(-1) + g(-1)$

$= 1 + -1 = \boxed{0}$

26) $(f \circ g)(-2) = f(g(-2))$

$= f(-2)$

$= 0$

27) $(g \circ f)(2) = \cancel{f}g(f(2))$

$= g(0)$

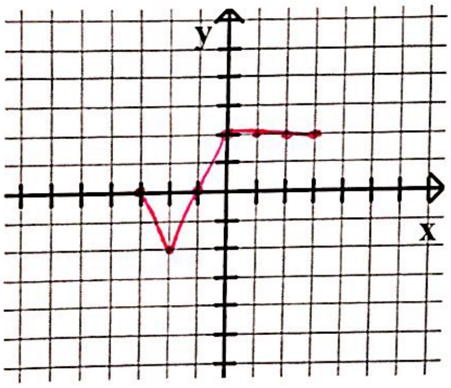
$= \boxed{0}$

28) $(f+g)(3)$

$= f(3) + g(3)$

$= -1 + 3 = \boxed{2}$

EX) Use the table to graph $(f+g)(x)$



29)

x	f	g	f + g
-3	1	-1	0
-2	0	-2	-2
-1	1	-1	0
0	2	0	2
1	1	1	2
2	0	2	2
3	-1	3	2

ordered pair

(-3, 0)
(-2, -2)
(-1, 0) etc.

EX) The number of bacteria in a refrigerated food is given by $N(T) = 20T^2 - 80T + 500, \quad 2 \le T \le 14$ where T is the Celsius temperature of the food. When the food is removed from refrigeration, the temperature is given by $T(t) = 4t + 2, \quad 0 \le t \le 3$ where t is the time in hours. Find the following:

30) The composite N [T (t)]. What does this function represent?

\# of bacteria at a given time

$N(T(t)) = N(4t+2) = 20(4t+2)^2 - 80(4t+2) + 500$

$= \cancel{320} 320t^2 + 320t + 80 - 320t - 160 + 500 = \underline{320t^2 + 420}$

31) The number of bacteria in the food when t = 2 hours.

$N(T(2)) = 320(2)^2 + 420 = \underline{1700 \text{ bacteria}}$

32) Use your graphing calculator to find the time when the bacteria count reaches 2000.

$t = 2.22$ hours