1-6: INVERSE FUNCTIONS

☺ OBJECTIVES: Find inverse functions and graph them. Determine if functions are one-to-one.

INVERSE FUNCTIONS

The inverse of a function f is denoted by $f^{-1}(x)$. The inverse of a function is the equation, values, or graph that makes the domain become the range and the range become the domain.

The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}. $f^{-1}(x) \neq \frac{1}{f(x)}$. Be careful of the symbolism! * Domain & Range switch

Since f and f^{-1} have the effect of "undoing" each other, $f\left[f^{-1}(x)\right] = f^{-1}\left[f(x)\right] = x$.

* You may be asked to find the inverse of a function *graphically, by looking at ordered pairs, or by being given an equation.*

To find an inverse function you:

① Switch x & y

② Solve for y

Find the inverses of the following functions. List the domain and range of both the original and the inverse.

Verify that $f(f^{-1}(x)) = x$.

1. $f(x) = 4x - 3$

$x = 4y - 3$

$x + 3 = 4y$

$\frac{x+3}{4} = y$

$\boxed{f^{-1}(x) = \frac{x+3}{4}}$

$f(f^{-1}(x)) = 4\left(\frac{x+3}{4}\right) - 3$

$= x + \cancel{3} - \cancel{3}$

$= \boxed{x}$ ✓

f: D: ℝ, R: ℝ

f^{-1}: D: ℝ, R: ℝ

2. $f(x) = (x+2)^3$

$x = (y+2)^3$

$\sqrt[3]{x} = y + 2$

$\sqrt[3]{x} - 2 = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x} - 2}$

f	f^{-1}
D: ℝ	D: ℝ
R: ℝ	R: ℝ

$f(f^{-1}(x)) = (\sqrt[3]{x} - \cancel{2} + \cancel{2})^3$

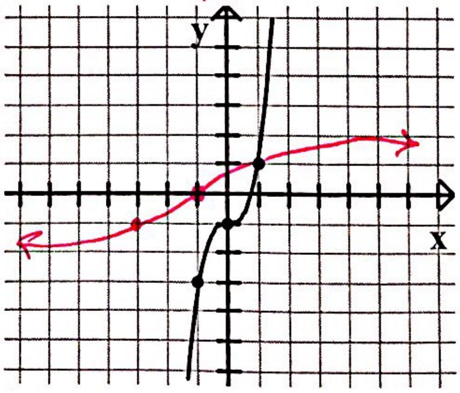
$= \sqrt[3]{x}^{\,3}$

$= x$ ✓

GRAPHS OF AN INVERSE FUNCTIONS

For a given function, $y = f(x)$, you sketch a graph of the inverse by simply switching the x and y values of the coordinates of $f(x)$. Or you can graph the new inverse equation by reflecting the line $y = x$. This reflection switches x & y places for you ☺

The graph of $f(x) = 2x^3 - 1$ is shown at right.



3) Draw the inverse by hand.

4) Find the equation of the inverse of $f(x)$ by hand.
Check on your calculator to see if your graph is correct.

$y = 2x^3 - 1$

$x = 2y^3 - 1$

$\frac{x+1}{2} = \cancel{2}y^3$

$\sqrt[3]{\frac{x+1}{2}} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}}$ ✓

THE EXISTENCE OF AN INVERSE FUNCTION

A function may not have an inverse which is a *function*. If *both* the function *and* its inverse are functions (they both pass the vertical line test), we call this a **one-to-one function.** No 2 elements in the domain of f correspond to the same element in the range of f. (No repeating x's or y's.)

* A one-to-one function is both a function of x and a function of y. Therefore its graph will pass both the vertical line test and the horizontal line test.

* *For continuous functions to be one-to-one, they must be either strictly increasing or strictly decreasing.*

Example: Which functions below are one-to-one and have an inverse that is a function?

5) $f(x)=3x+1$ 6) $g(x)=x^2-x$ 7) $h(x)=2^x$ 8) $h(x)=x^3+1$

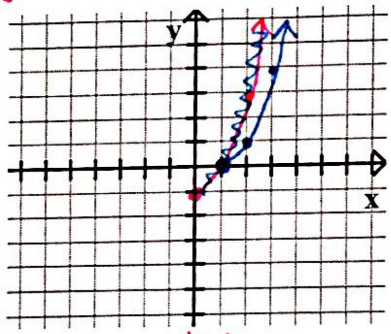
One-to-one

not one-to-one

one-to-one

one-to-one

9) Restrict the domain of $f(x)=(x-1)^2$ so it will be one-to-one, find the inverse and graph both f and f^{-1}.



*restriction is generally to cut off left-hand side of graph

Restriction: ~~$x \geq 0$~~ $x \geq 1$

now it's one-to-one

Examples: Find the inverse (if it exists) for the following. List the domain and range of both the original function and its inverse. *Be careful... you may need to include a domain restriction.

Verify your answer for #10 by showing that $g\left(g^{-1}(x)\right)=g^{-1}\left(g(x)\right)=x$.

*make sure graphs are inverses

10) $g(x)=\sqrt{2x-3}$ (only "+" roots)

$x=\sqrt{2y-3}$

$x^2=2y-3$

$x^2+3=2y$

$\frac{x^2+3}{2}=y$

$\boxed{\frac{1}{2}x^2+\frac{3}{2}=y,\ x\geq 0}$

*original function is only "+" roots so graph is

Need to restrict inverse so only half a parabola like function is

11) $h(x)=x^2+1,\ x\leq 0$

Graph

$x=y^2+1$

$x-1=y^2$

$\pm\sqrt{x-1}=y$

graph

*need to restrict. cut off top half to match f(x).

$\boxed{f^{-1}(x)=-\sqrt{x-1}}$