Unit 8
Radical Functions

Day 1

Operations with Functions

You have seen that functions can be combined by adding, subtracting, multiplying and dividing.
The domain of the new function consists of the x-values that are in the domains of both f and g. There may be additional restrictions for quotient functions so that the denominator is not 0.

Function Operations

Addition: $(f+g)(x)=f(x)+g(x)$

Subtraction: $(f-g)(x)=f(x)-g(x)$

Multiplication: $(f\cdot g)(x)=f(x)\cdot g(x)$

Division: $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}; g(x)\neq 0$

Composition of Functions

A composite function is formed when the output from one function (inner) becomes the input for another function (outer). The composition of function f with function g is written as $(f\circ g)(x)$ or $f(g(x))$.

Domain: look at inside functions domain and combine it with the answer's domain.

Examples.

Let $f(x)=3x+8$ and $g(x)=x^2-5$.

1. Find the new function and the domain:

a. $(f+g)(x)$

$= 3x+8+x^2-5$

$= \boxed{x^2+3x+3}$

$\boxed{\mathbb{R}}$

b. $(f-g)(x)$

$= 3x+8-(x^2-5)$

$= 3x+8-x^2+5$

$= \boxed{-x^2+3x+13}$

$\boxed{\mathbb{R}}$

c. $(f\circ g)(x)$

$= f(g(x))$

$= f(x^2-5)$

$= 3(x^2-5)+8$

$= 3x^2-15+8$

$= \boxed{3x^2-7}$

D: inside $\downarrow$ $\mathbb{R}$ answer $\mathbb{R}$ so D: $\boxed{\mathbb{R}}$

2. Find the quantity:

a. $\left(\frac{f}{g}\right)(3) = \frac{f(3)}{g(3)}$

$= \frac{3(3)+8}{(3)^2-5}$

$= \frac{9+8}{9-5} = \boxed{\frac{17}{4}}$

b. $(f\circ g)(-1)$

$= f(g(-1))$

$= f((-1)^2-5)$

$= f(1-5)$

$= f(-4)$

$= 3(-4)+8 = -12+8 = \boxed{-4}$

c. $(f\cdot g)(0)$

$= f(0)\cdot g(0)$

$= 3(0)+8\cdot(0)^2-5$

$= 8\cdot(-5)$

$= \boxed{-40}$

Let $f(x)=x^2-4$ and $g(x)=\sqrt{x+3}$.

3. Find the new function and its domain:

a. $(g\cdot f)(x)$

$\boxed{(\sqrt{x+3})\cdot(x^2-4)}$

D: $\boxed{x\geq -3}$

b. $\left(\frac{g}{f}\right)(x)$

$= \boxed{\frac{\sqrt{x+3}}{x^2-4}}$

D: $x^2-4\neq 0$

$x^2\neq 4$

$\boxed{x\neq 2,-2}$

$x+3\geq 0$

$\boxed{x\geq -3}$

c. $f(g(x))$

$= f(\sqrt{x+3})$

$= (\sqrt{x+3})^2-4$

$= x+3-4$

$= \boxed{x-1}$

D: inside $\downarrow$ $x\geq -3$ Answers $\downarrow$ $\mathbb{R}$ so D: $\boxed{x\geq -3}$

4. Find the quantity:

a. $f(g(1))$

$= f(\sqrt{1+3})$

$= f(\sqrt{4})$

$= f(2)$

$= (2)^2-4$

$= \boxed{0}$

b. $(f+g)(-4)$

$= f(-4)+g(-4)$

$= (-4)^2-4+\sqrt{-4+3}$

$= 16-4+\sqrt{-1}$

$= 12+i$ $\boxed{\text{not real}}$

c. $(g\circ f)(3)$

$= g(f(3))$

$= g(3^2-4)$

$= g(5)$

$= \sqrt{5+3} = \sqrt{8} = \boxed{2\sqrt{2}}$

5. Two functions f and g are graphed below. Find the following quantities from the graphs:

a. $(f+g)(2) = f(2) + g(2)$
$= 3 + 3$
$= \boxed{6}$

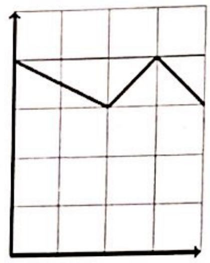
b. $(g-f)(3) = g(3) - f(3)$
$= 0 - 4$
$= \boxed{-4}$

c. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{4}{2} = \boxed{2}$

d. $(f \cdot g)(4) = f(4) \cdot g(4)$
$= 3 \cdot 2$
$= \boxed{6}$

e. $(f \circ g)(2) = f(g(2))$
$= f(3)$
$= \boxed{4}$

f. $g(f(3)) = g(4)$
$= \boxed{2}$



$f(x)$

$g(x)$

Assignment 8.1

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Day 2

Inverse Functions

The **inverse function** of f, symbolized by f^{-1}, is the function containing all of the ordered pairs of f with the coordinates switched. For example, if (a, b) is contained in f, then (b, a) is contained in f^{-1}. This means that the domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}.

NOTE:
$f^{-1}(x) \neq \frac{1}{f(x)}$

Example:

1. List the domain and range for the function and its inverse: $f(x)$ {(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)}

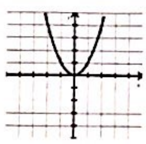
$f(x)$ domain: {0, 1, 2, 3, 4} range: {0, 2, 4, 6, 8}

$f^{-1}(x)$ domain:: {0, 2, 4, 6, 8} range: {0, 1, 2, 3, 4}

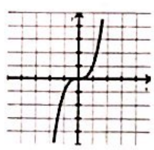
In order for the inverse function to exist, f must be **one-to-one** (no two elements in the domain of f can correspond to the same element in the range of f). A function is one-to-one if its graph passes the **horizontal line test**, meaning ***no*** horizontal line can be drawn that passes through the graph of f more than once.

To be a function it must pass the **vertical line test**. To be a function with an inverse it must also pass the **horizontal line test**. If it passes **BOTH** the vertical and horizontal line tests then it's a **one-to-one function**.

Examples. Determine whether each function is one-to one.

2.  $f(x) = x^2$

no

3.  $f(x) = x^3$

yes

4. $f(x) = x^4 + x^2$

 no

If you do not know whether a function is one-to-one, graph it.

Finding an Inverse	**1. Write the function in the form** $y = f(x)$ **2. Switch** x **and** y. **3. Solve for** y. **4. Replace** y **with** $f^{-1}(x)$

Examples. Find the inverse of each of the following one-to-one functions:

5. $f(x) = 5x - 3$

$y = 5x - 3$

$x = 5y - 3$

$\frac{x+3}{5} = \frac{5y}{5}$

$y = \frac{x+3}{5}$

$\boxed{f^{-1}(x) = \frac{x+3}{5}}$

6. $f(x) = \frac{2x-6}{7}$

$y = \frac{2x-6}{7}$

$7x = \frac{2y-6}{7} \cdot 7$

$7x = 2y - 6$

$7x + 6 = 2y$

$y = \frac{7x+6}{2}$

$\boxed{f^{-1}(x) = \frac{7x+6}{2}}$

7. $f(x) = \frac{x}{x-3}$

$y = \frac{x}{x-3}$

$(y-3)x = \frac{y}{y-3}(y-3)$

$xy - 3x = y - \cancel{xy}$

$-3x = y - xy$

$\frac{-3x}{1-x} = \frac{y(1-x)}{1-x}$

$y = \frac{-3x}{1-x}$

$\boxed{f^{-1}(x) = \frac{-3x}{1-x}}$

8. $f(x) = (x-2)^3$

$y = (x-2)^3$

$x = (y-2)^3$

$\sqrt[3]{x} = y - 2$

$\sqrt[3]{x} + 2 = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x} + 2}$

9. $f(x) = \sqrt[3]{2x} + 5$

$y = \sqrt[3]{2x} + 5$

$x = \sqrt[3]{2y} + 5$

$(x-5)^3 = (\sqrt[3]{2y})^3$

$\frac{(x-5)^3}{2} = \frac{2y}{2}$

$\boxed{f^{-1}(x) = \frac{(x-5)^3}{2}}$

Verify Inverses

If two functions are inverses, the following is true: $f\left(f^{-1}(x)\right) = x$ AND $f^{-1}\left(f(x)\right) = x$

Thus, you can verify that two functions are inverses of each other by "composing" them both ways.

Example:

10. Show that $f(x) = 5x - 2$ and $g(x) = \frac{x+2}{5}$ are inverses of each other.

$f(g(x))$

$= f\left(\frac{x+2}{5}\right)$

$= 5\left(\frac{x+2}{5}\right) - 2$

$= x + \cancel{2} - \cancel{2}$

$= \boxed{x}$ ✓

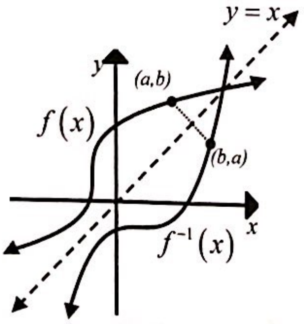
$g(f(x))$

$= g(5x - 2)$

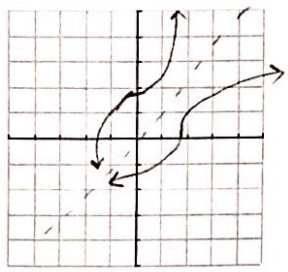
$= \frac{5x - 2 + 2}{5}$

$= \frac{5x}{5} = \boxed{x}$ ✓

The graph of f^{-1} is the reflection of the graph of f across the line $y = x$. Thus, for (a, b) on the graph of f, (b, a) will be on the graph of f^{-1}.

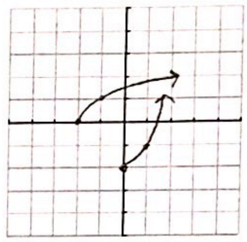


Graph f and g in a graphing calculator along with the line $y = x$ to "visually verify" that $f(x) = \frac{1}{2}x^3 + 2$ and $g(x) = \sqrt[3]{2x - 4}$ are inverses.



Examples.

11. Graph $f(x) = \sqrt{x+2}$.



a. Does $f(x) = \sqrt{x+2}$ pass the horizontal line test? (Is it one-to-one?)

yes

b. Find the domain and range for $f(x) = \sqrt{x+2}$ from the graph.

domain: $x \geq -2$ range: $y \geq 0$

c. Find the inverse of $f(x) = \sqrt{x+2}$.

$y = \sqrt{x+2}$

$x^2 = \sqrt{y+2}^{\,2}$

$x^2 - 2 = y$

$\boxed{f^{-1}(x) = x^2 - 2}$

d. Do you have to specify a domain for $f^{-1}(x)$? If so, what is that domain?

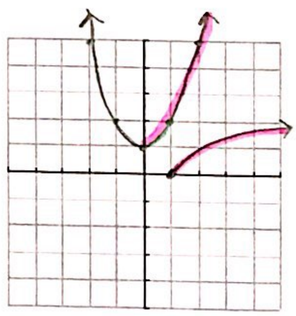
yes, f^{-1} has to be one-to-one so $x \geq 0$

e. Graph $f^{-1}(x)$ in the same coordinate plane as $f(x)$. What do you notice?

Reflection of each other

12. Graph the function $f(x) = x^2 + 1$ and find its inverse.
When a function is NOT one-to-one, the domain must be restricted so that the inverse will be a function.
Choose all the x-values to the right of the vertex.

Domain: $x \geq 0$



$y = x^2 + 1$

$x = y^2 + 1$

$\pm\sqrt{x-1} = \sqrt{y^2}$

$\pm\sqrt{x-1} = y$

Only +

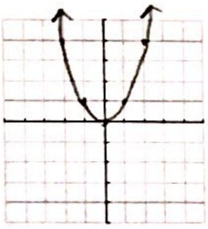
$f^{-1}(x) = \sqrt{x-1}$

Day 3

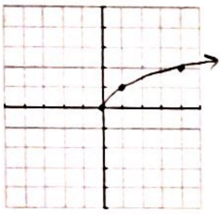
Graphing Radical/Absolute Value Functions

Five parent functions are given below. Graph each function.

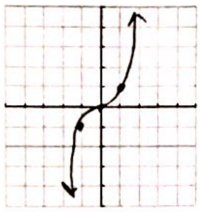
$f(x)=x^2$



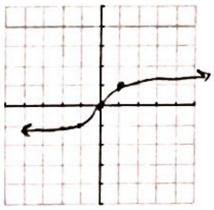
$f(x)=\sqrt{x}$



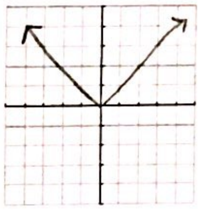
$f(x)=x^3$



$f(x)=\sqrt[3]{x}$



$f(x)=|x|$



Using these graphs, many other graphs can be produced using shifts, dilations (stretch/compress), and reflections of the original graph.

A Summary of Graphing Adjustments

Vertical Shifts $f(x) \pm c$ up $+c$ down $-c$	**Horizontal Shifts*** $f(x \pm c)$ left $+c$ Right $-c$
Vertical Stretch/Compression $a f(x)$ $\lvert a\rvert > 1$ stretch $\lvert a\rvert < 1$ Shrink compression	**Horizontal Stretch/Compression*** $f(ax)$ $\lvert a\rvert > 1$ Shrink $\lvert a\rvert < 1$ Stretch
Reflection across *x*-axis $-f(x)$ reflect across x axis	**Reflection across *y*-axis*** $f(-x)$ reflect across y axis

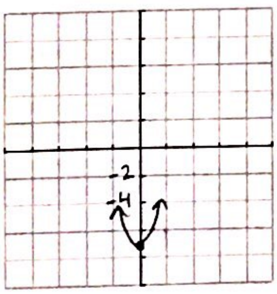
*Be really careful with adjustments to x-values, as they do the opposite of what they say.

order of transformations: reflections/dilations then shifts

Examples. State the transformations and graph each of the following. State the domain and range of each.

1. $f(x)=x^2-7$

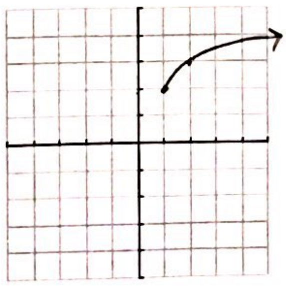
Down 7



2. $g(x)=\sqrt{x-1}+2$

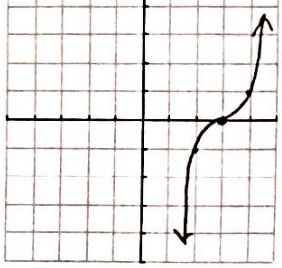
up 2

Right 1



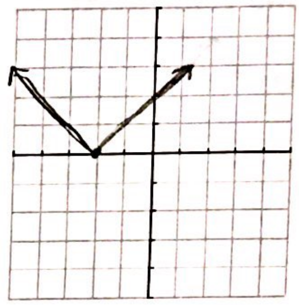
3. $h(x) = (x-3)^3$

Right 3



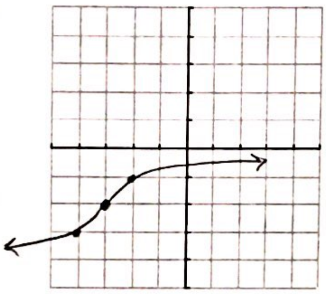
4. $f(x) = |x+2|$

left 2



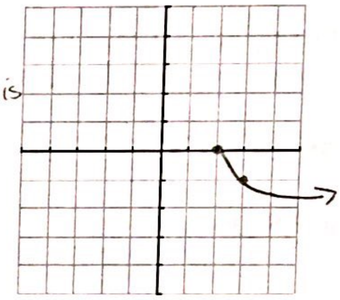
5. $f(x) = \sqrt[3]{x+3} - 2$

Down 2
Left 3



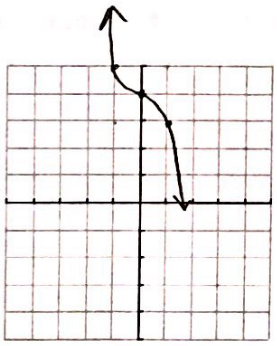
6. $g(x) = -\sqrt{x-2}$

reflect x axis
right 2



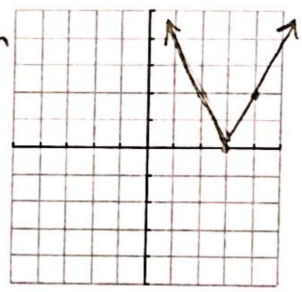
7. $h(x) = -x^3 + 4$

reflect x axis
up 4



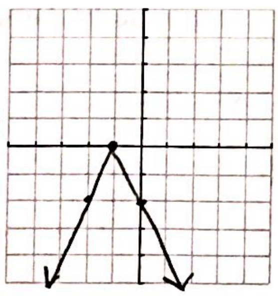
8. $f(x) = 2|x-3|$

Vertical Stretch by 2
right 3



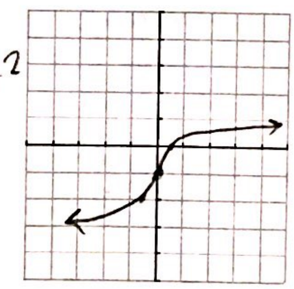
9. $f(x) = -2|x+1|$

reflect x axis
vertical stretch 2
Left 1



10. $f(x) = \sqrt[3]{2x} - 1$

Horizontal Shrink 2
Down 1



Day 4

Extracting Roots and Rewriting Radicals

Review: Simplify.

$10^2 \cdot 10^3 = 10^5$ $(10^2)^3 = 10^6$ $(10^3)^2 = 10^6$ $\frac{10^6}{10^2} = 10^4$ $\frac{10^2}{10^6} = \frac{1}{10^4}$ $\frac{10^2}{10^2} = 1$

The following **properties of exponents** work for any base. (The base above is 10.)

1. $x^m \cdot x^n = x^{m+n}$
2. $\frac{x^m}{x^n} = x^{m-n}$
3. $(x^m)^n = x^{mn}$
4. $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
5. $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$
6. $x^0 = 1,\ x \neq 0$ (0^0 is undefined.)

Examples. Simplify. (Don't leave answers with zero or negative exponents.)

1. $(-3ab^4)(4ab^{-3})$

$= -12a^2b$

2. $(2xy^{-2})^3$

$= 2^3x^3y^{-6}$

$= \boxed{\frac{8x^3}{y^6}}$

3. $\frac{(2ab^2)^3}{6a^2b^{-3}}$

$= \frac{8a^3b^6}{6a^2b^{-3}}$

$= \boxed{\frac{4ab^9}{3}}$

Radicals and Rational Exponents

$\sqrt[n]{x} = x^{\frac{1}{n}}$ $\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}} = (\sqrt[n]{x})^m$ $64^{\frac{2}{3}} = \sqrt[3]{64^2} = (\sqrt[3]{64})^2 = 4^2 = 16$

Examples.

4. Write each rational exponent as a radical.

a. $19^{1/2}$

$= \sqrt[2]{19^1} = \sqrt{19}$

b. $11^{3/4}$

$= \sqrt[4]{11^3}$

c. $14x^{2/3}$

$= \sqrt[3]{14x^2}$

d. $x^{-5/7}$

$= \frac{1}{\sqrt[7]{x^5}}$

5. Write each radical as an exponential expression.

a. $\sqrt{37}$

$= 37^{1/2}$

b. $\sqrt[4]{9^3}$

$= 9^{3/4}$

c. $(\sqrt[8]{x})^2$

$= x^{\frac{2}{8}} = \boxed{x^{\frac{1}{4}}}$

6. Simplify.

a. $7^{1/2}\left(7^{1/4}\right)$ add $\frac{1(2)}{2(2)}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4}=\frac{3}{4}$

$= \boxed{7^{\frac{3}{4}}}$

b. $\frac{3^{2/3}}{3^{7/3}}$ subtract $\frac{2}{3}-\frac{7}{3}=-\frac{5}{3}$

$= 3^{-5/3}$

$= \frac{1}{3^{5/3}}$

c. $\frac{\left(x^{1/2}y^{3/4}\right)^2}{y^{3/2}}$

$= \frac{x\,y^{6/4}}{y^{3/2}} = \frac{x\,y^{3/2}}{y^{3/2}} = \boxed{x}$

Simplify Radicals

Simplifying a radical means to remove as much of the quantity as possible from inside the radical (radicand).

> The following **properties of radicals** follow from the properties of exponents:
>
> $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ $\qquad$ $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}$

Examples.

7. Simplify.

a. $\sqrt{24}$

2 12

2 4

2 3

$= \boxed{2\sqrt{6}}$

b. $\sqrt{108}$

2 54

2 27

3 9

3 3

$= \boxed{6\sqrt{3}}$

c. $\sqrt[3]{16}$

2 8

2 4

2 2

$= \boxed{2\sqrt[3]{2}}$

d. $\sqrt[4]{-162}$

Not Real

e. $\sqrt{\frac{7}{36}}$

$= \frac{\sqrt{7}}{\sqrt{36}}$

$= \boxed{\frac{\sqrt{7}}{6}}$

8. Multiply or divide first, then simplify.

a. $\sqrt[3]{3}\cdot\sqrt[3]{16}$

$= \sqrt[3]{48}$

2 24

2 12

2 6

2 3

$= \boxed{2\sqrt[3]{6}}$

b. $\sqrt{7}\cdot\sqrt{14}$

$= \sqrt{98}$

2 49

7 7

$= \boxed{7\sqrt{2}}$

c. $\frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36}$

$= \boxed{6}$

9. Simplify. (Sometimes it helps to write the expression in rational exponent form, apply properties of exponents and then rewrite in radical form):

a. $\sqrt[9]{5^6} = 5^{6/9}$

$= 5^{2/3}$

$= \boxed{\sqrt[3]{5^2}}$

b. $\sqrt[4]{2}\cdot\sqrt[5]{2}$ $\qquad \frac{1(5)}{4(5)}+\frac{1(4)}{5(4)}=\frac{9}{20}$

$= 2^{1/4}\cdot 2^{1/5}$

$= 2^{9/20} = \boxed{\sqrt[20]{2^9}}$

c. $\sqrt{x}\cdot\sqrt[3]{x}$ $\qquad \frac{1(3)}{2(3)}+\frac{1(2)}{3(2)}=\frac{5}{6}$

$= x^{1/2}\cdot x^{1/3}$

$= x^{5/6}$

$= \boxed{\sqrt[6]{x^5}}$

10. Simplify. Assume that all variables are positive.

a. $\sqrt{16x^3}$

$= \boxed{4x\sqrt{x}}$

b. $\sqrt[3]{8x^4y^5}$

$= \boxed{2xy\sqrt[3]{xy^2}}$

c. $-\sqrt{32y^9}$

$= \boxed{-4y^4\sqrt{2y}}$

d. $\sqrt[3]{2xy^2} \cdot \sqrt[3]{14x^2y^2}$

$= \sqrt[3]{28x^3y^4}$

$= \boxed{xy\sqrt[3]{28y}}$

e. $\sqrt[5]{\frac{x}{32}}$

$= \frac{\sqrt[5]{x}}{\sqrt[5]{32}}$

$= \boxed{\frac{\sqrt[5]{x}}{2}}$

f. $-\sqrt[3]{\frac{m^6}{125}}$

$= -\frac{\sqrt[3]{m^6}}{\sqrt[3]{125}}$

$= \boxed{-\frac{m^2}{5}}$

Assignment 8.4
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Day 5

Operations with Radicals (11.5)

Add/Subtract Radicals

When **adding or subtracting radicals**, the root index AND the radicands must be the same. Radicals must often be simplified before they can be combined.

Examples. Simplify and combine (if possible). Assume that all the variables are positive.

1. $\sqrt{72} + \sqrt{32}$

36 2

$= 6\sqrt{2} + 4\sqrt{2} = \boxed{10\sqrt{2}}$

2. $2\sqrt{18} - \sqrt{27} + 3\sqrt{12} - \sqrt{2}$

9 2 9 3 4 3

$= 6\sqrt{2} - 3\sqrt{3} + 6\sqrt{3} - \sqrt{2}$

$= \boxed{5\sqrt{2} + 3\sqrt{3}}$

3. $3\sqrt[3]{81} + \sqrt[3]{24}$

27 3 2 12 2 6 2 3

$= 9\sqrt[3]{3} + 2\sqrt[3]{3} = \boxed{11\sqrt[3]{3}}$

4. $5\sqrt{32x} + 4\sqrt{98x}$

16 2 49 2

$= 20\sqrt{2x} + 28\sqrt{2x} = \boxed{48\sqrt{2x}}$

5. $\boxed{14\sqrt{x} + 3\sqrt{y}}$

can't combine because they aren't same

6. $7\sqrt[3]{x^2} - 2\sqrt[3]{x^2}$

$= \boxed{5\sqrt[3]{x^2}}$

7. $x\sqrt[3]{16x} - \sqrt[3]{54x^4}$

8 2 27 2

$= 2x\sqrt[3]{2x} - 3x\sqrt[3]{2x}$

$= \boxed{-x\sqrt[3]{2x}}$

Rationalize Denominators

Sometimes it's necessary to **remove radicals from denominators** of fractions. This process is called **rationalizing.** It's very similar to obtaining a common denominator.

- To rationalize a square root denominator, multiply the **numerator and denominator** of the fraction by the square root in the denominator.
- If the denominator contains a cube root, you must actually think about what you should multiply by, so that the denominator is no longer a radical.

Examples. Rationalize the denominators:

8. $\frac{5}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{6\cdot 3} = \boxed{\frac{5\sqrt{3}}{18}}$

9. $\frac{2}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \boxed{\frac{2\sqrt[3]{25}}{5}}$

10. $\frac{6x^2}{\sqrt{8x}} \cdot \frac{\sqrt{8x}}{\sqrt{8x}} = \frac{6x^2\sqrt{8x}}{8x} = \frac{12x^2\sqrt{2x}}{8x}$

$= \boxed{\frac{3x\sqrt{2x}}{2}}$

11. $\frac{2\sqrt[3]{x}}{\sqrt[3]{2x^2}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \frac{2\sqrt[3]{4x^5}}{2x^2}$

$= \frac{x\sqrt[3]{4x^2}}{x^2}$

$= \boxed{\frac{\sqrt[3]{4x^2}}{x}}$

Assignment 8.5

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Day 6

Solving Radical Equations

To solve radical equations, **you need to "undo" the radical.** To undo a square root, square both sides; to undo a cube root, cube both sides; etc.

- **Isolate** a single radical on one side of the equation **first.**
- When you square or cube an equation, you square/cube **sides, not terms.**
- Squaring both sides can create **extraneous solutions. Check your solutions** in the original equation.

Examples. Solve for *x*.

1. $\sqrt{2x-1}^{\,2} = 3^2$

$2x - 1 = 9$

$2x = 10$

$x = \frac{10}{2}$

$\boxed{x = 5}$

Check!

$\sqrt{2(5)-1} \stackrel{?}{=} 3$

$\sqrt{10-1} \stackrel{?}{=} 3$

$3 = 3$ ✓

2. $\left(\sqrt[3]{\frac{x}{2}+1}\right)^3 = -2^3$

$\frac{x}{2} + 1 = -8$
$\quad -1$

$\frac{x}{2} = -9$

$x = -18$

Check

$\sqrt[3]{\frac{-18}{2}+1} \stackrel{?}{=} -2$

$\sqrt[3]{-9+1} \stackrel{?}{=} -2$

$\sqrt[3]{-8} = 2$ ✓

3. $\frac{\sqrt[3]{3-2x}}{2}+\cancel{3}=4-3$

$2\cdot\frac{\sqrt[3]{3-2x}}{\cancel{2}} = 1\cdot 2$

$\sqrt[3]{3-2x}^{\,3} = 2^3$

$\cancel{3}-2x = 8-3$

-3

$-2x = 5$

$\boxed{x = -\frac{5}{2}}$

Check

$\frac{\sqrt[3]{3-2(-\frac{5}{2})}}{2} + 3 \stackrel{?}{=}$

$\frac{\sqrt[3]{3+5}}{2} + 3$

$\frac{\sqrt[3]{8}}{2} + 3$

$\frac{2}{2} + 3$

$1+3 = 4$ ✓

4. $(x-3)^2 = \sqrt{x+3}^{\,2}$

$\boxed{\text{FOIL}}$

$x^2-6x+9 = x+3$

$x^2-7x+6 = 0$

$(x-6)(x-1) = 0$

$\boxed{x=6}$ ~~$x=1$~~ extraneous

Check

$6-3 \stackrel{?}{=} \sqrt{6+3}$

$3 = 3$ ✓

$1-3 \stackrel{?}{=} \sqrt{1+3}$

$-2 \neq 2$

Solving equations with rational exponents

When x is inside an expression with a rational exponent, you can raise both sides of the equation to a reciprocal power to get rid of the exponent. Be sure to isolate the expression FIRST.

Examples. Solve for x.

5. $(x+1)^{\cancel{3/2}} = 8^{2/3}$

$x+1 = \sqrt[3]{8}^{\,2}$

$x+1 = 2^2$

$x+1 = 4$

$\boxed{x=3}$

6. $(x-1)^{2/3} - \cancel{4} = 5+4$

$(x-1)^{\cancel{2/3}} = 9^{3/2}$

$x-1 = \sqrt{9}^{\,3}$

$x-1 = 3^3$

$x-1 = 27$

$\boxed{x=28}$

7. The speed that a tsunami (tidal wave) can travel is modeled by the equation $S = 356\sqrt{d}$ where S is the speed in kilometers per hour and **d** is the average depth of the water in kilometers.

a. What is the speed of the tsunami when the average water depth is 0.512 kilometers?

$S = 356\sqrt{.512} = \boxed{255 \text{ kmph}}$

b. Solve the equation for d.

$\frac{S}{356} = \frac{356\sqrt{d}}{356}$

$\left(\frac{S}{356}\right)^2 = \sqrt{d}^{\,2}$

$\boxed{d = \frac{S^2}{356^2}}$

c. A tsunami is measured traveling at 120 kilometers per hour. What is the average depth of the water?

$d = \frac{120^2}{356^2} = \boxed{.114 \text{ Km}}$

Assignment 8.6

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Day 7

Unit 8 Review

Assignment 8.7

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Day 8

Unit 8 Test ***All late/absent assignments due for Unit 8***

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