Practice Test / Unit 9 Key **Secondary 3 Honors**

Simplify the following. Make sure to show all work.

1. $\tan\theta\cot\theta - \cos^2\theta$

$= \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} - \cos^2\theta$

$> 1 - \cos^2\theta$

$= \boxed{\sin^2\theta}$

2. $\cos\theta\sec\theta - \frac{\cos\theta}{\sec\theta}$

$= \cos\theta\cdot\frac{1}{\cos\theta} - \frac{\cos\theta}{\frac{1}{\cos\theta}}$

$= 1 - \cos\theta\cdot\frac{\cos\theta}{1}$

$= 1 - \cos^2\theta = \boxed{\sin^2\theta}$

3. Find the <u>exact value</u> of $\cos 75^\circ$ using a <u>sum or difference</u> identity.

$\cos 75^\circ$

$= \cos(30^\circ + 45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

4. Find the exact value of $\sin(u+v)$ given than $\sin u = -\frac{15}{17}$ and $\cos v = \frac{5}{13}$.

Both u and v are in Quadrant IV.

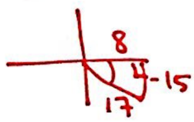
$\sin(u+v) = \sin u \cos v + \sin v \cos u$

$= -\frac{15}{17}\left(\frac{5}{13}\right) + \frac{-12}{13}\left(\frac{8}{17}\right)$

$= \frac{-75}{221} - \frac{96}{221}$

$= \boxed{\frac{-171}{221}}$

u =



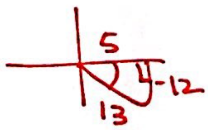
$x^2 + (-15)^2 = 17^2$

$x^2 = 64$

$x = \pm 8 = +8$

$\cos u = \frac{8}{17}$

v =



$y^2 + 5^2 = 13^2$

$y^2 = 144$

$y = \pm 12$

$= -12$

$\sin v = -\frac{12}{13}$

5. Use a <u>double-angle</u> formula to find the exact value of $\sin 2u$ when $\cos u = -\frac{12}{13}$, where $\pi < u < \frac{3\pi}{2}$. QIII

-12, -5, 13 $\quad \sin u = \frac{-5}{13}$

$\sin 2u = 2\sin u \cos u$

$= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right) = \boxed{\frac{120}{169}}$

6. Use the <u>half-angle</u> identities to find the exact value of $\cos 67.5°$. QI so $\pm$

$\cos\left(\frac{135°}{2}\right) = \sqrt{\frac{1+\cos 135°}{2}} = \sqrt{\frac{1+-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

Solve the following equations for <u>all values</u> of x.

7. $2\sqrt{2} = -4\cos\theta$

$\frac{2\sqrt{2}}{-4} = \frac{-4\cos\theta}{-4}$ $\quad$ ref∠ $= \pi/4$

$-\frac{\sqrt{2}}{2} = \cos\theta$

$\boxed{\theta = \frac{3\pi}{4} + 2\pi n, \ \frac{5\pi}{4} + 2\pi n}$

8. $2\cot\theta + \cot^2\theta = -1$

$\cot^2\theta + 2\cot\theta + 1 = 0$

$(\cot\theta + 1)(\cot\theta + 1) = 0$

$\cot\theta + 1 = 0$

$\cot\theta = -1$

$(\tan\theta = -1)$ $\quad$ ref∠ $= \pi/4$

$\boxed{\theta = \frac{3\pi}{4} + 2\pi n, \ \frac{7\pi}{4} + 2\pi n}$

Solve the following equations in the interval $[0, 2\pi)$.

9. $2\sin 3\theta + 1 = 0$

$\sin 3\theta = -\frac{1}{2}$ $\quad$ ref∠ $= \pi/6$

$\frac{3\theta}{3} = \frac{\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}}{3}$

$\boxed{\theta = \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}, \frac{35\pi}{18}}$

10. $\cos^2\theta + 2 = -3\cos\theta + \sin^2\theta$

$\cos^2\theta + 2 = -3\cos\theta + 1 - \cos^2\theta$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$2\cos\theta + 1 = 0 \qquad \cos\theta + 1 = 0$

$\cos\theta = -1/2 \qquad \cos\theta = -1$

$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi}$

$(-1, 0)$

ref∠ $= \pi/3$

Verify the following identities.

11. $\tan^2\alpha\sec^2\alpha+\sec^2\alpha=\sec^4\alpha$

$= \sec^2\alpha(\tan^2\alpha+1)$

$= \sec^2\alpha(\sec^2\alpha)$

$= \underline{\underline{\sec^4\alpha}}$

12. $\dfrac{\sec\theta\sin\theta}{\tan\theta+\cot\theta}=\sin^2\theta$

$= \dfrac{\frac{1}{\cos\theta}\cdot\sin\theta}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$

$= \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta\cos\theta}}$

$= \dfrac{\sin\theta}{\cancel{\cos\theta}}\cdot\dfrac{\sin\theta\cancel{\cos\theta}}{1}$

$= \underline{\sin^2\theta}$