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Unit 10 Notes / Secondary 3 Honors

Day 1: Operations with Radicals & Exponents

RATIONAL EXPONENTS: $a^{\frac{m}{n}} = \sqrt[n]{a}^{\,m} = \sqrt[n]{a^m}$

Rewrite the expression using rational exponent notation.

1. $\sqrt[3]{12} = \boxed{12^{1/3}}$
2. $\left(\sqrt[4]{10}\right)^7 = \boxed{10^{7/4}}$
3. $\sqrt[5]{(x+7)^2} = \boxed{(x+7)^{2/5}}$

Rewrite the expression using radical notation.

4. $5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$
5. $h^{\frac{3}{7}}g^{\frac{2}{7}} = \boxed{\sqrt[7]{h^3g^2}}$
6. $(x+8)^{\frac{2}{5}} = \boxed{\sqrt[5]{(x+8)^2}}$

Simplifying Radicals by extracting roots.

7. Simplify the radicals. Write your answer in reduced, radical form. Assume all variables are positive.

a. $\sqrt{48x^2y^3}$

$= \sqrt{16\cdot 3x^2y^2y}$

$= \boxed{4xy\sqrt{3y}}$

b. $\sqrt[3]{-54x^6y^2}$

$= \sqrt[3]{-27\cdot 2x^3x^3y^2}$

$= \boxed{-3x^2\sqrt[3]{2y^2}}$

c. $\sqrt{(x-3)^2}$

$= \boxed{(x-3)}$

Multiplying and Dividing Radicals:

$\sqrt[n]{a\bullet b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

8. Multiply or divide the radicals and write your answer in reduced, radical form. Assume all variables are positive.

a. $\sqrt{6x^3}\cdot\sqrt{12xy^2}$

$= \sqrt{72x^4y^2}$

$= \boxed{6x^2y\sqrt{2}}$

b. $\sqrt[3]{3x^2}\cdot\sqrt[3]{18x^5}$

$= \sqrt[3]{\underset{27\cdot 2}{54}x^7}$

$= \boxed{3x^2\sqrt[3]{2x}}$

c. $\sqrt{\frac{25x^3}{4x}} = \sqrt{\frac{25x^2}{4}}$

$= \boxed{\frac{5x}{2}}$

d. $\frac{\sqrt[3]{x}}{\sqrt[3]{2x}} = \sqrt[3]{\frac{x}{2x}} = \sqrt[3]{\frac{1}{2}}$

$= \frac{\sqrt[3]{1}}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{4}}{\sqrt[3]{4}}$

$= \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \boxed{\frac{\sqrt[3]{4}}{2}}$

same type of root w/ same value underneath

9. Add or subtract the radicals. Must be like terms to +/ −

a. $\sqrt{x-3}+\sqrt{x-3}$

$= \boxed{2\sqrt{x-3}}$

b. $2\sqrt{8}+3\sqrt{2}$

$= 2\cdot 2\sqrt{2}+3\sqrt{2}$

$= 4\sqrt{2}+3\sqrt{2}$

$= \boxed{7\sqrt{2}}$

c. $\sqrt[3]{24}-\sqrt[3]{3}$

$= \sqrt[3]{8\cdot 3}-\sqrt[3]{3}$

$= 2\sqrt[3]{3}-\sqrt[3]{3}$

$= \boxed{\sqrt[3]{3}}$

Simplify.

10. $m^{\frac{1}{2}}m^{\frac{1}{3}}$

$= \boxed{m^{5/6}}$

or $\sqrt[6]{m^5}$

11. $\frac{3}{x^{\frac{5}{2}}} = \frac{3}{\sqrt{x^5}}$

$= \frac{3}{x^2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$

$= \boxed{\frac{3\sqrt{x}}{x^3}}$

12. $\sqrt[4]{48x^2y^6z^{16}}$

$= \sqrt[4]{16\cdot 3\,x^2y^4y^2z^4z^4z^4z^4}$

$= \boxed{2yz^4\sqrt[4]{3x^2y^2}}$

13. $-6\sqrt[7]{2}+2\sqrt[7]{256}$

$= -6\sqrt[7]{2}+2\sqrt[7]{128\cdot 2}$

$= -6\sqrt[7]{2}+4\sqrt[7]{2}$

$= \boxed{-2\sqrt[7]{2}}$

14. $\sqrt[4]{y^9b^7}\cdot\sqrt[4]{y^3b^{10}}$

$= \sqrt[4]{y^{12}b^{17}}$

$= \boxed{y^3b^4\sqrt[4]{b}}$

15. $\frac{\sqrt{x^2w^5}}{\sqrt{x^7w^3}} = \sqrt{\frac{x^2w^5}{x^7w^3}}$

$= \sqrt{\frac{w^2}{x^5}}$

$= \frac{\sqrt{w^2}}{\sqrt{x^5}} = \frac{w}{x^2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$

$= \boxed{\frac{w\sqrt{x}}{x^3}}$

16. $\frac{-8\sqrt[3]{x}}{9\sqrt[4]{x}}$

$= \frac{-8x^{1/3}}{9x^{1/4}}$

$= \boxed{-\frac{8x^{1/12}}{9}}$ or $\frac{-8\sqrt[12]{x}}{9}$

17. $\sqrt[3]{108}\cdot\sqrt[3]{4}$

$= \sqrt[3]{27\cdot 4}\cdot\sqrt[3]{4}$

$= 3\sqrt[3]{4}\cdot\sqrt[3]{4}$

$= 3\sqrt[3]{16}$

$= \boxed{6\sqrt[3]{2}}$

18. $4\sqrt{p}(7\sqrt{p}-2\sqrt{p})$

$= 28p-8p$

$= \boxed{20p}$

19. $(j^{-3}p^9v^{-18})^{\frac{2}{3}}$

$= j^{-2}p^6v^{-12}$

$= \boxed{\frac{p^6}{j^2v^{12}}}$

20. $\sqrt{20}\cdot\sqrt{5}\cdot\sqrt{12}$

$= 4\sqrt{5}\cdot\sqrt{5}\cdot 2\sqrt{3}$

$= 4\cdot 5\cdot 2\sqrt{3}$

$= \boxed{40\sqrt{3}}$

21. $\left(\frac{5w^2y^{-3}}{-10w^7y}\right)^{-3}$

$= \left(\frac{1}{-2w^5y^4}\right)^{-3}$

$= (-2w^5y^4)^3$

$= \boxed{-8w^{15}y^{12}}$

Day 2: Solving Radical and Absolute Value Equations

STEPS TO SOLVE RADICAL EQUATIONS:

1) Isolate the radical on one side of the equation.
2) Raise each side of the equation to the same power to eliminate the radical.
3) Solve for the variable.
4) Check your solution. *Some solutions will be extraneous.*

Solve the radical equations for x.

1. $3\sqrt{x}+7=25$

$3\sqrt{x}=18$

$\sqrt{x}=6$

$\boxed{x=36}$

2. $\left(\sqrt[3]{2x-3}\right)^3=(2)^3$

$2x-3=8$

$2x=11$

$\boxed{x=11/2}$

3. $(x+5)^{\frac{2}{3}}+5=9$

$\sqrt[3]{x+5}^{\,2}+5=9$

$\sqrt[3]{x+5}^{\,2}=4$

$\sqrt[3]{x+5}=\pm 2$

$x+5=(\pm 2)^3$

$x+5=8 \qquad x+5=-8$

$\boxed{x=3 \qquad x=-13}$

4. $4\sqrt[5]{x}+5=-3$

$4\sqrt[5]{x}=-8$

$\sqrt[5]{x}=-2$

$\boxed{x=-32}$

5. $(3x+5)^{\frac{7}{3}}+22=150$

$\sqrt[3]{3x+5}^{\,7}=128$

$\sqrt[3]{3x+5}=2$

$3x+5=8$

$3x=3$

$\boxed{x=1}$

6. $(x-6)^2=\left(\sqrt{3x}\right)^2$

$x^2-12x+36=3x$

$x^2-15x+36=0$

$(x-12)(x-3)=0$

$\boxed{x=12} \qquad \cancel{x=3}$

7. $\sqrt{2x+7}-x=2$

$\left(\sqrt{2x+7}\right)^2=(x+2)^2$

$2x+7=x^2+4x+4$

$0=x^2+2x-3$

$=(x+3)(x-1)$

$\cancel{x=-3} \qquad \boxed{x=1}$

8. $\sqrt{2x+6}-\sqrt{x+4}=1$

$\left(\sqrt{2x+6}\right)^2=\left(1+\sqrt{x+4}\right)^2$

$2x+6=\left(1+\sqrt{x+4}\right)\left(1+\sqrt{x+4}\right)$

$2x+6=1+2\sqrt{x+4}+(x+4)$

$2x+6=x+5+2\sqrt{x+4}$

$(x+1)^2=\left(2\sqrt{x+4}\right)^2$

$x^2+2x+1=4(x+4)$

$x^2+2x+1=4x+16$

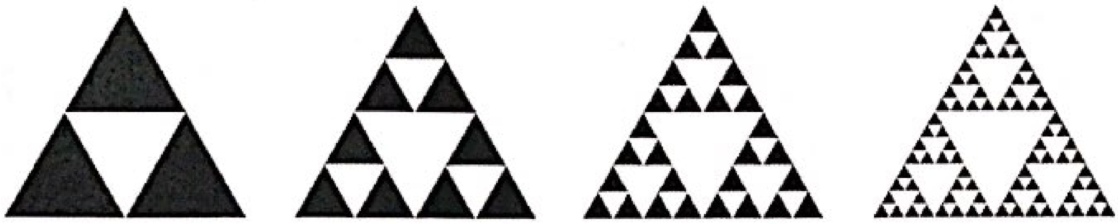
$x^2-2x-15=0$

$(x-5)(x+3)=0$

$\boxed{x=5} \qquad \cancel{x=-3}$

Day 3: Introduction to Sequences

Look for a pattern. Count the number of black triangles. Predict the number of black triangles if the pattern continues.



3 (3^1) 9 (3^2) 27 (3^3) 81 (3^4)

How many black triangles would be next in the pattern? $3^5 = \boxed{243}$

Can you write an equation that would describe the pattern with the triangles? $y = 3^n$

Sequence: **An ordered list of numbers for which there may be a pattern.**

An **infinite sequence** is a function whose domain is the set of positive integers and whose function values denoted by $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ are the **terms** of the sequence. If the domain of the function consists of the first n positive integers only, then the sequence is a **finite sequence**.

Example: Finite: 3, 9, 27, 81 Infinite: 3, 9, 27, 81 . . . . .

Finding terms in a sequence:

If a rule is given for a sequence, then the terms of the sequence can be generated from this rule.

Example: (Unless otherwise stated, assume a_1 begins with n = 1.)

1. Find the first four terms and the tenth term of the sequences defined by the given rules:

a. $a_n = 3n - 2$

$\boxed{1, 4, 7, 10 \quad a_{10} = 28}$

$a_1 = 3(1) - 2 = 1$

$a_2 = 3(2) - 2 = 4$

$a_3 = 3(3) - 2 = 7$

$a_4 = 3(4) - 2 = 10$

$a_{10} = 3(10) - 2 = 28$

b. $a_n = 3 + (-1)^n$

$\boxed{2, 4, 2, 4 \quad a_{10} = 4}$

$a_1 = 3 + (-1)^1 = 2$

$a_2 = 3 + (-1)^2 = 4$

$a_3 = 3 + (-1)^3 = 2$

$a_4 = 3 + (-1)^4 = 4$

$a_{10} = 3 + (-1)^{10} = 4$

Factorial Notation

If n is a positive integer, n factorial = n! = 1 • 2 • 3 • 4 •...• (n-1) • n

The TI-83 and 84 have a factorial operator in Math Prb 4. (Or try shortcut Alpha F2)

2. Evaluate:

a. 3! $= 3 \cdot 2 \cdot 1 = 6$

b. 5! $= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

c. 8! $= 8 \cdot 7 \cdot 6 \cdot 5 \cdots 1 = 40{,}320$

d. 0! $= \boxed{1}$

3. List the first 4 terms of the sequence $a_n = \dfrac{2^n}{n!}$

$a_1 = \dfrac{2^1}{1!} = 2$ $a_2 = \dfrac{2^2}{2!} = \dfrac{4}{2} = 2$ $a_3 = \dfrac{2^3}{3!} = \dfrac{8}{6} = \boxed{\dfrac{4}{3}}$ $a_4 = \dfrac{2^4}{4!} = \dfrac{16}{24} = \boxed{\dfrac{2}{3}}$

It is fairly simple to find the terms of a sequence when given a rule to find a_n. A tougher task is to try to find a rule for a_n when given terms of the sequence.

Patterns to look for:

Linear: $a_n = 3n+2$ (adding same # from term to term)

Exponential: $a_n = 3^n$ (multiplying same # from term to term)

Quadratic: $a_n = n^2+3$

Factorials: $a_n = \frac{1}{n!}$

4. Write expressions for a possible nth term of each sequence listed below:

a. 1, 3, 5, 7, ...

$\boxed{a_n = 2n-1}$

n	
1	1
2	3
3	5
4	7

b. 2, 5, 10, 17, ...

$\boxed{a_n = n^2+1}$

n	
1	2
2	5
3	10
4	17

c. $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$

$\boxed{a_n = \frac{(n+1)}{n}}$

n	top
1	2
2	3
3	4
4	5

n	bottom
1	1
2	2
3	3
4	4

d. 1, 1, 2, 6, 24, 120, ...

$\boxed{a_n = (n-1)!}$

n	
1	1
2	1
3	2
4	6
5	24
6	120

Recursively Defined Sequences

A sequence is defined recursively if it is defined in terms of previous terms. In such a case, you need to be given a previous term (or previous terms).

5. The Fibonacci Sequence is a famous example of a sequence which is recursively defined. Its terms are 1, 1, 2, 3, 5, 8, 13, 21, ...

The sequence can be written as $a_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$ ..... Yuck :-(!

Could also be written as $\boxed{a_{k+2} = a_k + a_{k+1}}$ OR $\boxed{a_n = a_{n-2} + a_{n-1}}$

6. List the 2nd, 3rd, 4th terms of $a_k = a_{k-1} + 2$ where $k \ge 2$ and $a_1 = 8$. (k−1) is term one before (k)

↳ "to find next term take term before and add 2"

$a_2 = a_1 + 2 = 8+2 = \boxed{10}$ $\quad a_4 = a_3 + 2 = 12+2 = \boxed{14}$

$a_3 = a_2 + 2 = 10+2 = \boxed{12}$

7. List the 3rd, 4th, and 5th terms of $a_{k+2} = a_k - 2a_{k+1} + 3$ and $a_1 = 5$, $a_2 = -7$.

$a_3 = a_1 - 2(a_2) + 3 = 5 - 2(-7) + 3 = \boxed{22}$

$a_4 = a_2 - 2(a_3) + 3 = -7 - 2(22) + 3 = \boxed{-48}$

$a_5 = a_3 - 2(a_4) + 3$
$= 22 - 2(-48) + 3$
$= \boxed{121}$

Summation (Sigma) Notation

$$\sum_{n=lower\ limit}^{upper\ limit} (formula\ to\ generate\ terms) = a_1 + a_2 + a_3 + \cdots + a_n$$

Note in the examples below that the lower limit does not need to be 1.

8. Find:

a. $\sum_{i=1}^{5} 3i$

$= 3(1) + 3(2) + 3(3) + 3(4) + 3(5)$

$= 3 + 6 + 9 + 12 + 15$

$= \boxed{45}$

b. $\sum_{k=3}^{6} (1+k^2)$

$= (1+3^2) + (1+4^2) + (1+5^2) + (1+6^2)$

$= 10 + 17 + 26 + 37 = \boxed{90}$

c. $\sum_{i=0}^{8} i!$

$= 0! + 1! + 2! + 3! + 4! + 5!$
$+ 6! + 7! + 8!$

$= \boxed{46{,}234}$

d. $\sum_{j=1}^{10} 5$

$= 5+5+5+5+5+5+5+5+5+5$

$= \boxed{50}$

9. Write an expression in sigma notation for the sum 5 + 8 + 11 + 14 + 17.

$$\boxed{\sum_{n=1}^{n=5} (3n+2)}$$

n	
1	5
2	8
3	11
4	14
5	17

$a_n = 3n+2$

Day 4: Arithmetic Sequences

An arithmetic sequence is a sequence whose consecutive terms have a **common difference.**

If we denote the common difference by d, then:

$a_2 - a_1 = d$

$a_3 - a_2 = d$

$a_4 - a_3 = d$

etc.

Examples:

$7, 11, 15, 19, \ldots \quad d = 4$

$2, -3, -8, -13, \ldots \quad d = -5$

$1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \ldots \quad d = \frac{1}{4}$

The n^{th} term of an arithmetic sequence has the form:

"start w/ first term add on how many common differences"

$\boxed{a_n = a_1 + d(n-1)}$ where a_1 = the value of the first term and d is the common difference.

Linear pattern

* Simplify answers

1. Find a formula for the n^{th} term of an arithmetic sequence whose first term is 2 and whose common difference is 3.

$a_1 = 2$
$d = 3$

$a_n = 2 + 3(n-1)$
$= 2 + 3n - 3$
$\boxed{a_n = 3n - 1}$

2. The 8^{th} term of an arithmetic sequence is 20, and the 17^{th} term is 65. Find the common difference d, and then find the first 3 terms of the sequence.

20 (8th) — — — — — — — — 65 (17th)
d d d . . . . .
9 d's between these
(17 − 8 = 9)

To find d:

$65 - 20 = 45$ total distance

$\frac{45}{9} = 5 = d$

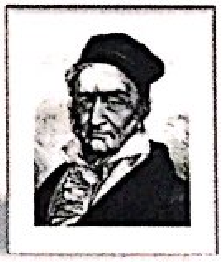
First term:

Start at 8th term & go back

$20 - 7(5) = -15$, First 3 terms $\boxed{-15, -10, -5}$

The SUM of a Finite Arithmetic Sequence

Listen to a story about a little boy . . . Carl Friedrich Gauss.



He was brilliant.....but he was naughty. His original problem was supposedly to find the sum of the integers from 1 to 100.

$S_n = 1 + 2 + 3 + \ldots + 100$

$1 + 100 = 101$
$2 + 99 = 101$
$3 + 98 = 101$
$4 + 97 = 101 \ldots$

50 pairs all add to 101

So total sum $= 50(101)$
$= \boxed{5050}$

The sum of an arithmetic sequence is given by the formula:

n = # of terms

$$\boxed{s_n = \frac{n}{2}(a_1 + a_n)}$$

Where n = number of terms being added, a_1 = value of first term, a_n = value of last term.

3. Find the sum of the first 150 terms of the sequence 5, 16, 27, 38, 49, ...

$a_n = 5 + 11(n-1)$

$S_{150} = \frac{150}{2}(5 + a_{150})$

$a_{150} = 5 + 11(149)$

$= 1644$

$S_{150} = \frac{150}{2}(5 + 1644)$

$= \boxed{123{,}675}$

4. An auditorium has 20 rows of seats. There are 30 seats in the first row, 31 seats in the second row, 32 seats in the third row, and so on. How many seats are there in all 20 rows?

$a_1 = 30$

$a_2 = 31$

$a_3 = 32$

$S_{20} = \frac{20}{10}(30 + a_{20})$

$= 10(30 + 49)$

$a_n = 30 + 1(d-1)$

$= \boxed{790 \text{ seats}}$

$a_{20} = 30 + 1(19) = 49$

5. A small business sells $10,000 worth of products during its first year. The owner of the business has a goal of increasing annual sales by $7500 each year for 9 years. Assuming that this goal is met, find the total sales during the first 10 years this business is in operation.

Sum

$a_1 = 10{,}000$

$d = 7500$

$a_{10} = 10{,}000 + 9(7500)$

$= \$77{,}500$

$S_{10} = \frac{10}{2}(10{,}000 + 77{,}500)$

$= \boxed{\$437{,}500}$

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Day 5: Geometric Sequences

A geometric sequence is a sequence whose consecutive terms have a common ratio.

If we denote the common ratio by r, then:

$$\frac{a_2}{a_1} = r,\ \frac{a_3}{a_2} = r,\ \frac{a_4}{a_3} = r,\ etc.\quad r \neq 0$$

Examples:

$2,\ 4,\ 8,\ 16, \ldots \quad r = 2$

$12,\ 36,\ 108,\ 324, \ldots \quad r = 3$

$\frac{-1}{3}, \frac{1}{9}, \frac{-1}{27}, \frac{1}{81}, \ldots \quad r = -\frac{1}{3}$

The n^{th} term of a geometric sequence has the form:

$$\boxed{a_n = a_1 (r)^{n-1}}$$

where a_1 = value of first term and r is the common ratio.

1. Write the first five terms of the geometric sequence whose first term (a_1) is 3 and whose common ratio (r) is 2.

$a_1 = 3$

$a_2 = 3(2) = 6$

$a_3 = 3(2)^2 = 12$

$\vdots$

$\boxed{3, 6, 12, 24, 48}$

2. Find the 12^{th} term of the geometric sequence 5, 15, 45, ...

$r = 3$

$a_1 = 5$

$a_n = 5(3)^{n-1}$

$a_{12} = 5(3)^{11} = \boxed{885{,}735}$

3. Assuming that the terms are positive, find the fourteenth term of a geometric sequence where $a_4 = 125$ and $a_{10} = \frac{125}{64}$.

$\underset{4th}{125}$ _ _ _ _ _ $\underset{10th}{125/64}$

r r r . . . .

6 "r's between them

$$\frac{\frac{125}{64}}{125} = \frac{125(r)^6}{125}$$

$\frac{1}{64} = r^6$

$\sqrt[6]{\frac{1}{64}} = r$

$r = \frac{1}{2}$

$a_4(r)^{10} = a_{14}$

$125(\frac{1}{2})^{10} = a_{14}$

$= \boxed{\frac{125}{1024}}$

The Sum of a Geometric Sequence

The sum of a geometric sequence is given by the formula:

$$\boxed{S_n = a_1\left(\frac{1-r^n}{1-r}\right)}$$

Finite sequence

4. Find the sum of the first 10 terms of the sequence 3, 2, $\frac{4}{3}$, $\frac{8}{9}$, ... $r = 2/3$

$$S_{10} = 3\left(\frac{1-\left(\frac{2}{3}\right)^{10}}{1-(2/3)}\right)$$

$$= \boxed{\frac{58025}{6561}} = 8.8439$$

Infinite Geometric Sequence

This is the formula for the sum of an infinite geometric sequence where $|r| < 1$

$$\boxed{S_\infty = \frac{a_1}{1-r}}$$

5. Find the sum of 3, 2, $\frac{4}{3}$, $\frac{8}{9}$, ... infinite

(as we add more and more terms, what the sum is approaching)

$$S = \frac{3}{1-2/3} = \boxed{9}$$

6. Find the sum $\sum_{n=1}^{\infty} 3(.2)^n$ infinite

* Make sure you know r

$a_1 = 3(.2)^1 = .6$

$a_2 = 3(.2)^2 = .12$ $\quad r = \frac{.024}{.12} = .2$

$a_3 = 3(.2)^3 = .024$

$$S = \frac{.6}{1-.2} = \boxed{.75}$$

Day 6: Applications of Sequences

1. A stomach virus spreads rapidly through a town. Initially only 12 people were infected, but the virus spreads quickly, increasing the number of people infected by 15% every day.

increase: 1 + %
decrease: 1 − %

a. Does this situation describe a geometric or arithmetic pattern?

geometric

b. How many new people are infected on the 10[th] day? $a_1 = 12$ $r = 1.15$

$$a_{10} = 12(1.15)^9 = 42.21 \approx \boxed{42 \text{ people}}$$

c. How many $\boxed{\text{total}}$ people were infected on the 10[th] day?

sum

Sum of 1st 10 terms

$$S_{10} = 12\left(\frac{1-1.15^{10}}{1-1.15}\right) = 243.64 \approx \boxed{243 \text{ people}}$$

2. Rhonda is considering two different physical therapist positions.

Range of Motion: offers an initial salary of $50,000 per year with an annual increase of $1,500 per year.

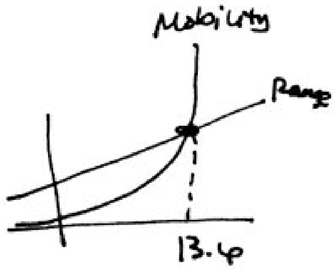
Mobility, Inc.: offers an initial salary of $42,000 with a guaranteed 4% increase in salary every year.

a. Which situation is arithmetic and which is geometric?

Range of Motion = Arithmetic

Mobility = Geometric

b. Determine the year for which Range of Motion pays more than Mobility, Inc.



Range

$a_n = 50{,}000 + 1500(n-1)$

$a_n = 48{,}500 + 1500n$

Mobility

$a_n = 42000(1.04)^{n-1}$

$n = 13.6$ $\boxed{13.6 \text{ years}}$

c. Determine which company pays more salary over a 30-year career.

Range

$a_{30} = 48{,}500 + 1500(29) = 93500$

$S_{30} = \frac{30}{2}(50000 + 93500) = \$2{,}152{,}500$

Mobility

$$\text{Sum}_{30} = 42000\left(\frac{1-1.04^{30}}{1-1.04}\right) = \boxed{\$2{,}355{,}569.39}$$

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3. Vince wants to purchase a laptop with high screen resolution for his gaming hobby. He charges the $1000 purchase to a credit card with 19% interest. The credit card company requires a minimum monthly payment of 2% of the balance on the card. He learns that when making a monthly payment, 75% of the minimum payment goes toward interest and the remaining portion goes toward paying off the principal. Vincent decides he can only pay the minimum payment each month. Calculate the monthly payment details for the first 6 months. The first month is already shown in the table.

Number of Months (n)	Balance Before Monthly Payment ($)	2% Minimum Monthly Payment ($)	25% Amount Paid Toward Principal ($)	75% Amount Paid Toward Interest ($)	Balance After Monthly Payment ($)
1	1000.00	20.00	5.00	15.00	995.00
2	995.00	19.90	4.98	14.92	990.02
3	990.02	19.80	4.95	14.85	985.07
4	985.07	19.70	4.93	14.77	980.14
5	980.14	19.60	4.90	14.70	975.24
6	975.24	19.50	4.88	14.62	970.36

4. Look at the data in the second to last column (Amount paid toward interest). Determine whether the sequence of numbers follows a geometric or arithmetic pattern. Find the nth term formula for the sequence.

Sequence: 15, 14.92, 14.85, 14.77, 14.70 14.62 $r = .995$ (remember we rounded #'s)

Arithmetic or Geometric?

$a_n = 15(.995)^{n-1}$ General nth term formula.

5. How much money will Vince have paid in interest by the end of 12 months? total amount

$$S_{12} = 15\left(\frac{1-.995^{12}}{1-.995}\right) = \boxed{\$175.13}$$

6. Look at the data in the last column (balance after monthly payment). Determine whether the sequence of numbers follows a geometric or arithmetic pattern. Find the nth term formula for the sequence.

geometric $r = .995$

$$\boxed{a_n = 995(.995)^{n-1}}$$

7. What will Vince's balance be at the end of 12 months? $a_{12} = 995(.995)^{11} = \boxed{\$941.62}$