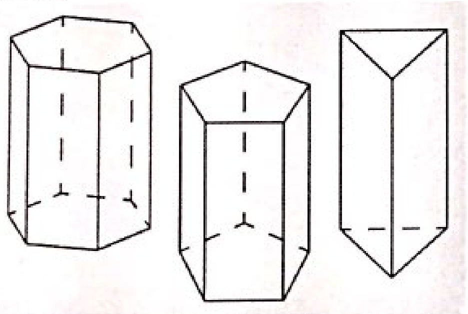
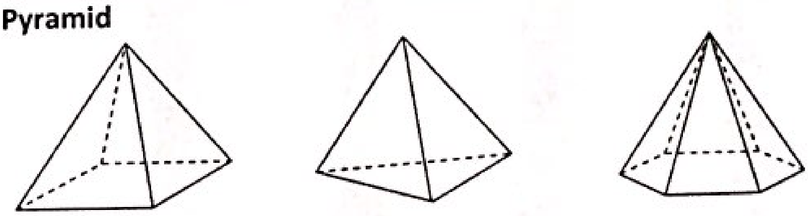
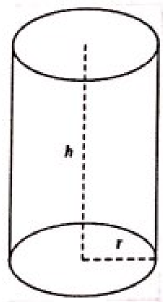
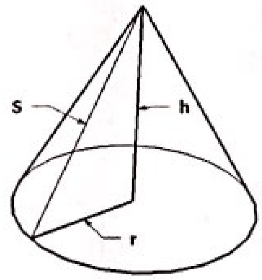
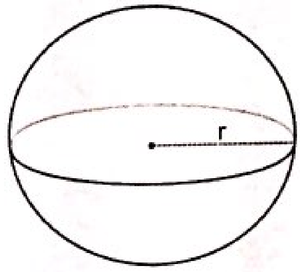
Teacher Copy

Unit 11 Notes / Secondary 3 Honors

Day 1: 3D Figures

3-dimensional figures:

Prism	**Pyramid**	
		
Cylinder	**Cone**	**Sphere**
		

Surface Area: The total area of the surface of a 3-dimensional object. (If you have a box of cereal, the surface area would be the sum of the areas of the top, bottom, front, back, left, and right sides of the box.)

In order to find surface area, you need to know how to find the areas of different shapes.

Area Formulas:

Rectangle = base x height

Triangle = $\frac{1}{2}$ x base x height

Circle = πr^2

Trapezoid = $\frac{1}{2}(b_1 + b_2)h$

Volume: The amount of space a 3-dimensional object takes up.

Volume Formulas:

Prism = $(area\ of\ base)(height)$

Pyramid = $\frac{1}{3}(area\ of\ base)(height)$

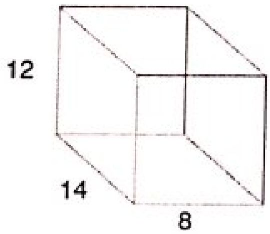
Cylinder = $(\pi r^2)(height)$

Cone = $\frac{1}{3}(\pi r^2)(height)$

Sphere = $\frac{4}{3}\pi r^3$

Examples:
Find the surface area and the volume of the following:

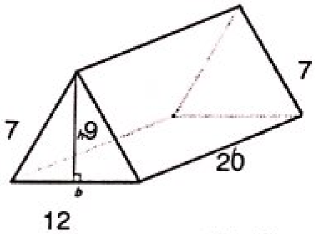
1. Surface Area = $752\ u^2$
   Volume = $1344\ u^3$



$SA = 2(14)(12) + 2(8)(14) + 2(12)(8)$
$= 752u^2$

$V = (12)(14)(8) = 1344u^3$

2. Surface Area = $628\ u^2$
   Volume = $1080\ u^3$

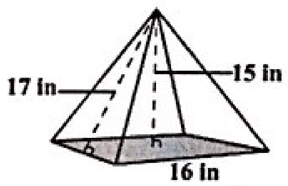


$SA = 2\left(\frac{1}{2}(12)(9)\right) + 2(20)(7) + 20(12)$
$= 628u^2$

$V = \frac{1}{2}(12)(9)(20) = 1080u^3$

3. Surface Area = $800\ in^2$
   Volume = $1280\ in^3$

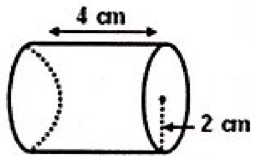
Hint: the base is a square



$SA = 16(16) + 4\left(\frac{1}{2}(16)(17)\right) = 800\ in^2$

$V = \frac{1}{3}(16)(16)(15) = 1280\ in^3$

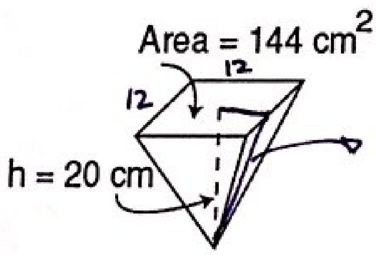
4. Surface Area = $75.4\ cm^2$
   Volume = $50.27\ cm^3$

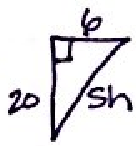


$SA = 2(\pi(2)^2) + 4(2\pi 2) = 75.40\ cm^2$

$V = \pi(2)^2(4) = 50.27\ cm^3$

5. A company produces 100,000 square based pyramid shaped popcorn containers every day, such as the one in the diagram below. (The top of the popcorn container is open so you can fill it with popcorn.) Calculate the total amount of paper the company uses for its popcorn containers in one day. (HINT: you will need to solve for the slant height.... Remember the Pythagorean theorem?)





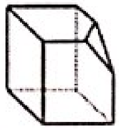
$sh = \sqrt{20^2 + 6^2}$
$= 20.88cm$

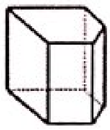
$SA = 4\left(\frac{1}{2}\right)(12)(20.88)$
$= 102.24\ cm$
for one container

100,000 containers –
$100{,}000(102.24)$
$= \boxed{100{,}224{,}000\ cm^2}$

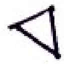
Cross Sections:

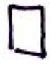
Students in Mrs. Denton's class were given cubes made of clay and asked to slice off a corner of the cube with a piece of dental floss.

Jumal sliced his cube this way. 

Jabari sliced his cube like this. 

A **cross section** is the face formed when a three-dimensional object is sliced by a plane. It can also be thought of as the intersection of a plane and a solid.

1. Draw and describe the cross section formed when Jumal sliced his cube. triangle 

2. Draw and describe the cross section formed when Jabari sliced his cube.  rectangle

3. Describe some other possible cross-sections that can be formed when a cube is sliced by a plane. pentagon, square, hexagon

How can you determine the largest polygon that can result from a plane intersecting a solid?

# of faces = largest cross section polygon

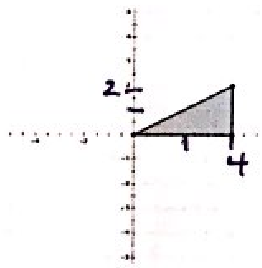
Solids of Revolution:

Perhaps you have used a pottery wheel or played with a spinning top or watched a figure skater spin so rapidly she looked like solid blur. The clay bowl, the rotating top, and the spinning skater can each be modeled as **solids of revolution** – a three-dimensional object formed by spinning a two-dimensional figure about an axis.

Suppose the right triangle shown below is rotating rapidly about the x-axis. Like the spinning skater, a solid image would be formed by the blur of the rotating triangle.

4. Draw and describe the solid of revolution formed by rotating this triangle about the x-axis.

Cone 



5. Find the volume of the solid formed.

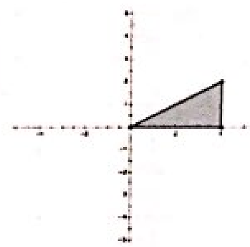
$h=4$

$r=2$

$V=\frac{1}{3}(\pi(2)^2)(4) = \boxed{16.76\ u^2}$

6. What would this figure look like if the triangle rotates rapidly about the y-axis?
   Draw and describe the solid of revolution formed by rotating this triangle about the y-axis.

cylinder with cone cut out of the top

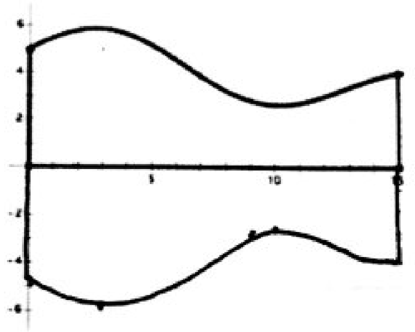


7. Find the volume of the solid formed.

Cylinder volume − Cone volume

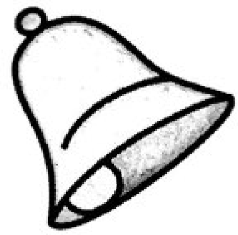
$= 2(\pi(2)^2(4)) - \frac{1}{3}(\pi(2)^2(4)) = \boxed{67.02\ u^3}$

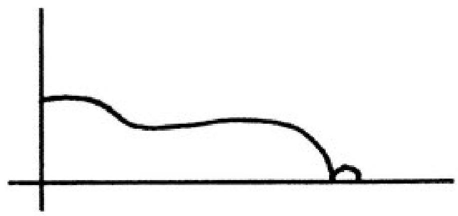
8. What about the following two-dimensional figure? Draw and describe the solid of revolution formed by rotating this figure about the x-axis.

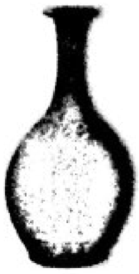


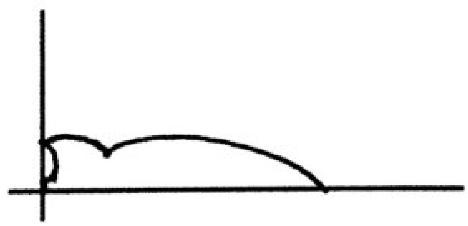
→ like a vase

9. For each of the following solids, draw the two-dimensional shape that generates it.









Day 2: **Solving Systems of Equations**

System of Equations: two or more equations that are solved together

The solution of a system is:

- Values of the variables that make <u>ALL</u> equations in the system true.
- points of intersection for the graphs

A system of ***linear*** equations can have how many solutions? 0, 1, infinitely many

Example. Solve the system.

1. $-5\begin{cases} (x-y=0) \\ 5x-3y=10 \end{cases}$

$-5x+5y=0$
$5x-3y=10$
$2y=10$
$y=5$

$x-5=0$
$x=5$

$\boxed{\begin{matrix} x=5 \\ y=5 \end{matrix}}$

How can you tell if the system has no solution or infinitely many solutions?

* No Solution – Variables cancel out – false statement (0=5)

* Infinite Solution – Variables cancel out – true statement (5=5

Solving Non-Linear Systems

When two lines intersect, it is at a single point (if it's an independent system). But what about a line and a parabola? What about two circles? When non-linear systems are solved, there is often more than one solution (point of intersection).

Possible Number of Points of Intersection			
	Line	Parabola	Circle
Line	0, 1, infinite	0, 1, 2	0, 1, 2
Parabola	0, 1, 2	0, 1, 2, 3, 4 infinite	0, 1, 2, 3, 4
Circle	0, 1, 2	0, 1, 2, 3, 4	0, 1, 2, infinite

Non-linear systems can be solved using substitution or elimination.

Examples.

2. Solve by substitution

$$\begin{cases} x^2+3x-y=1 \\ -2x+y=5 \end{cases} \qquad y=5+2x$$

$x^2+3x-(5+2x)=1$

$x^2+3x-5-2x=1$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \mid x=2$

$y=5+2(-3) = -1 \qquad y=5+2(2) = 9$

$\boxed{(-3,-1)} \qquad \boxed{(2,9)}$

3. Solve by elimination

$$\begin{cases} 3x^2+2y^2=36 \\ 2(4x^2-y^2=4) \end{cases}$$

$3x^2+2y^2=36$

$8x^2-2y^2=8$

$11x^2=44$

$x^2=4$

$x=\pm 2$

$x=2$: $4(2)^2-y^2=4$, $-y^2=-12$, $y^2=12$, $y=\pm 2\sqrt{3}$

$x=-2$: $4(-2)^2-y^2=4$, $y=\pm 2\sqrt{3}$

$\boxed{(2,2\sqrt{3}),\ (2,-2\sqrt{3}),\ (-2,2\sqrt{3}),\ (-2,-2\sqrt{3})}$

4. Solve

$$-\begin{cases} (-x+y=4) \\ x^2+y=3 \end{cases}$$

$x-y=-4$

$x^2+y=3$

$x^2+x=-1$

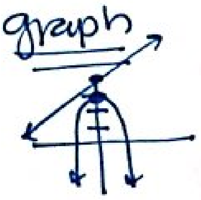
$x^2+x+1=0$

$x=\dfrac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}$

$=\dfrac{-1\pm\sqrt{-3}}{2}$ i

$\boxed{\text{No Solution}}$

graph



5. Solve

$$\begin{cases} -(x^2+y^2=13) \\ x^2-y=7 \end{cases}$$ → circle → parabola

$-\cancel{x^2}-y^2=-13$

$\cancel{x^2}-y=7$

$-y^2-y=-6$

$0=y^2+y-6$

$0=(y+3)(y-2)$

$y=-3$: $x^2-(-3)=7$, $x^2=4$, $x=\pm 2$

$y=2$: $x^2-2=7$, $x^2=9$, $x=\pm 3$

$\boxed{(2,-3),\ (-2,-3),\ (3,2),\ (-3,2)}$

6. The sum of two numbers is 7 and the sum of their squares is 29. Find the numbers.

$x+y=7 \qquad x=7-y$

$x^2+y^2=29$

$(7-y)^2+y^2=29$

$49-14y+y^2+y^2=29$

$2y^2-14y+20=0$

$y^2-7y+10=0$

$(y-5)(y-2)=0$

$y=5,\ y=2$

$x=7-5 \qquad x=7-2$

$x=2 \qquad x=5$

$\boxed{\text{\#'s are 2 and 5}}$

$(2,5)$

$(5,2)$

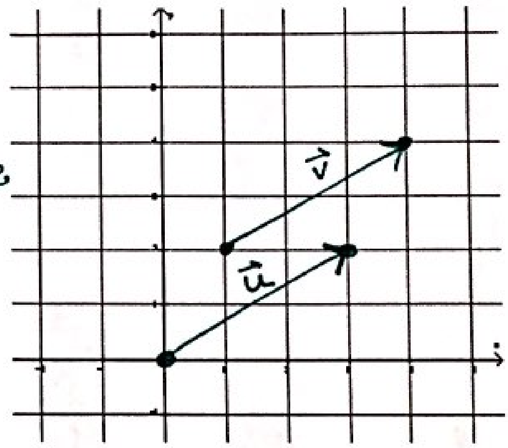
Day 3: Vectors in a Plane

* In order to have a vector, or a directed line segment, you need two things: *Direction* and *Magnitude* (length) represented by the symbol $\|\vec{v}\|$

* Vectors are denoted by lowercase, boldface letters such as **u, v,** and **w**. But since we can't write them as boldface letters we will use $\vec{u}, \vec{v}, \vec{w}$.

1. Let $\vec{u}$ be represented by the directed line segment from $\overset{\text{initial}}{P=(0,0)}$ to $\overset{\text{terminal}}{Q=(3,2)}$ and let $\vec{v}$ be represented by the directed line segment from $\overset{\text{initial}}{R=(1,2)}$ to $\overset{\text{terminal}}{S=(4,4)}$. Show that $\vec{u}=\vec{v}$. (Show that both vectors have the same direction and magnitude (length))

First, let's graph each line segment.
Second, find the magnitude or length of each.



$$\left.\begin{aligned} \|\vec{u}\| &= \sqrt{(3-0)^2+(2-0)^2} = \sqrt{9+4} = \sqrt{13} \\ \|\vec{v}\| &= \sqrt{(4-1)^2+(4-2)^2} = \sqrt{9+4} = \sqrt{13} \end{aligned}\right\} \text{same magnitude}$$

Third, find the slope of each.

$$\left.\begin{aligned} \text{Slope of } \vec{u} &= \frac{2-0}{3-0} = \frac{2}{3} \\ \text{Slope of } \vec{v} &= \frac{4-2}{4-1} = \frac{2}{3} \end{aligned}\right\} \text{same direction}$$

So it follows that $\vec{u}=\vec{v}$.

* When a vector has its **initial point** at the origin the vector is in **standard position.** A vector in standard position can be uniquely represented by the coordinates of its terminal point (v_1, v_2). This is called **component form** of a vector v, and is written as: $\vec{v}=\langle v_1, v_2\rangle$

V_1 is horizontal distance from start
V_2 is vertical distance from start

* **Component Form of a Vector:**

The component form of the vector with initial point $P=(p_1,p_2)$ and terminal point $Q=(q_1,q_2)$ is given by:

Keep this in order -- always end - start

* $\overline{PQ}=\langle q_1-p_1, q_2-p_2\rangle=\langle v_1,v_2\rangle=\vec{v}$ (x end - x start, y end - y start) or (x_2-x_1, y_2-y_1)

* The magnitude or length is: $\|\vec{v}\|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(v_1)^2+(v_2)^2}$

* Two vectors $\vec{u}=\langle u_1,u_2\rangle$ and $\vec{v}=\langle v_1,v_2\rangle$ are equal if and only if $u_1=v_1$ and $u_2=v_2$.

Find the component form and magnitude of vector **v** that has an initial point P and terminal point Q.

2. P=(4 , - 7), Q=(- 1 , 5)

Component form:

$v_1 = -1 - 4 = -5$

$v_2 = 5 - {}^{-}7 = 12$

$\boxed{\vec{v} = \langle -5, 12 \rangle}$

Magnitude:

$\|\vec{v}\| = \sqrt{(-5)^2 + (12)^2} = \sqrt{25+144} = \sqrt{169}$
$= \boxed{13}$

3. P=(- 3 , 5), Q=(2 , 6)

Component form:

$v_1 = 2 - {}^{-}3 = 5$

$v_2 = 6 - 5 = 1$

$\boxed{\vec{v} = \langle 5, 1 \rangle}$

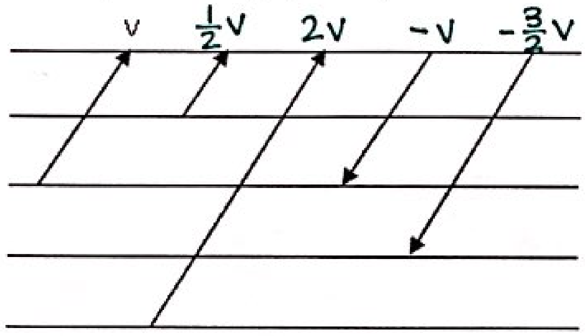
Magnitude:

$\|\vec{v}\| = \sqrt{5^2 + 1^2} = \sqrt{25+1}$
$= \boxed{\sqrt{26}}$

*** Vector Operations:**

Scalar Multiplication is represented graphically below. Finish labeling the vectors.

#

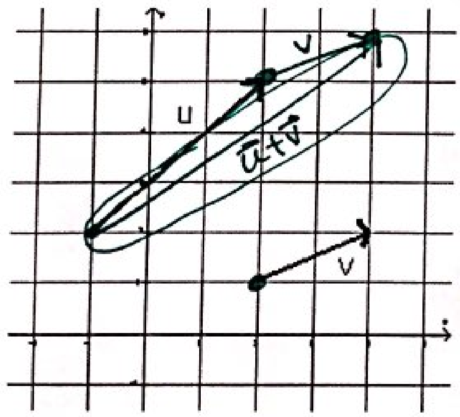


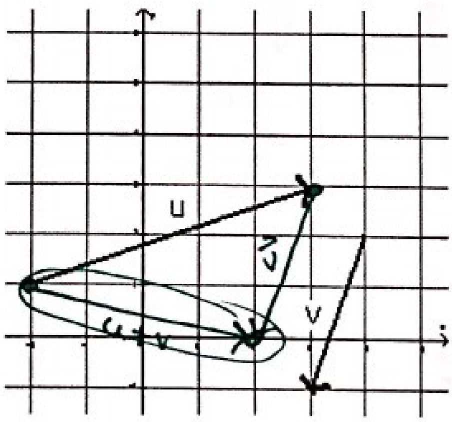
Vector Addition or the **resultant** vector: Can be represented by drawing a parallelogram.

* or the "tip to tail" method ☺

- connect 2nd vector to the end of the first vector. $\vec{u} + \vec{v}$ is the vector from the start of $\vec{u}$ to the end of $\vec{v}$

4. Add $\vec{u}+ \vec{v}$ and draw the resultant vector.





$\vec{u}$ in component form: $\langle 3, 3 \rangle$

$\vec{v}$ in component form: $\langle 2, 1 \rangle$

$\vec{u}+\vec{v}$ in component form: $\langle 5, 4 \rangle$

$\vec{u}$ in component : $\langle 5, 2 \rangle$

$\vec{v}$ in component: $\langle -1, -3 \rangle$

$\vec{u}+\vec{v}$ in component: $\langle 4, -1 \rangle$

Definitions:

Let $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ be vectors and k be a scalar.

Then the sum of **u** and **v** is the vector: **u** + **v** = $\langle u_1 + v_1, u_2 + v_2 \rangle$

And the scalar multiple of k times **u** is the vector: k**u** = $k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$

The negative of $\vec{v} = \langle v_1, v_2 \rangle$ is: -**v** = $-\langle v_1, v_2 \rangle = \langle -v_1, -v_2 \rangle$

The difference of **u** and **v** is the vector: **u** - **v** = **u** + (-**v**) = $\langle u_1 - v_1, u_2 - v_2 \rangle$

Let **v** = $\langle -2, 5 \rangle$ and **w** = $\langle 3, 4 \rangle$. Find each of the following vectors.

5. 2**v**

$= 2\langle -2, 5 \rangle$

$= \boxed{\langle -4, 10 \rangle}$

6. **w** − **v**

$= \langle 3, 4 \rangle - \langle -2, 5 \rangle$

$= \boxed{\langle 5, -1 \rangle}$

7. **v** + 2**w**

$= \langle -2, 5 \rangle + 2\langle 3, 4 \rangle$

$= \langle -2, 5 \rangle + \langle 6, 8 \rangle$

$= \boxed{\langle 4, 13 \rangle}$

8. 2**v** − 3**w**

$= 2\langle -2, 5 \rangle - 3\langle 3, 4 \rangle$

$= \langle -4, 10 \rangle - \langle 9, 12 \rangle$

$= \boxed{\langle -13, -2 \rangle}$

Unit Vectors:

In many applications of vectors, it is useful to find a unit vector. To do this, you divide **v** by its magnitude or length to obtain:

u = unit vector = $\dfrac{\vec{v}}{\|\vec{v}\|} = \left(\dfrac{1}{\|\vec{v}\|}\right)\vec{v}$

Linear combination form:

Another way to write a vector is the form used in Physics.

The x-coordinate is $\vec{i}$ or the horizontal component of $\vec{v}$.

The y-coordinate is $\vec{j}$ or the vertical component of $\vec{v}$.

In math we call this form a linear combination of the vectors **i** and **j**.

So given $\vec{v} = \langle 3, 5 \rangle$ = 3i + 5j

mean the same thing ☺

Find a unit vector in the direction of the given vector:

9. v = $\langle -2,5\rangle$

$\|\vec{v}\| = \sqrt{(-2)^2+(5)^2} = \sqrt{29}$

Unit Vector

$\frac{\langle -2,5\rangle}{\sqrt{29}} = \langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\rangle$

$= \boxed{\langle \frac{-2\sqrt{29}}{29}, \frac{5\sqrt{29}}{29}\rangle}$

10. u = $\langle 5,12\rangle$

$\|u\| = \sqrt{5^2+12^2}$

$= 13$

unit vector

$\frac{\langle 5,12\rangle}{13}$

$= \boxed{\langle \frac{5}{13}, \frac{12}{13}\rangle}$

11. w = $4\vec{i} - 3\vec{j}$

$= \langle 4,-3\rangle$

$\|\vec{w}\| = \sqrt{16+9} = 5$

Unit Vector

$\frac{\langle 4,-3\rangle}{5} = \boxed{\langle \frac{4}{5}, \frac{-3}{5}\rangle}$

12. Let **u** be the vector with initial point (2 ,- 5) and terminal point (- 1, 3). Write **u** as a linear combination of the standard unit vectors **i** and **j**.

$V_1 = -1-2 = -3$

$V_2 = 3 - {}^-5 = 8$

$\langle -3,8\rangle = \boxed{-3i+8j}$

13. Let **u** = -3i + 8j and **v**= 2i – j. Find 2u – 3v.

$\langle -3,8\rangle$ $\langle 2,-1\rangle$

$2u-3v = 2\langle -3,8\rangle - 3\langle 2,-1\rangle$

$= \langle -6,16\rangle + \langle -6,3\rangle$

$= \langle -12,19\rangle = \boxed{-12i+19j}$

14. Find the vector **v** with the given magnitude and same direction as **u**. $\|\vec{v}\| = 7$ and $\vec{u} = 3\vec{i} + 4\vec{j}$

Find unit vector for $\vec{u}$.

$\|\vec{u}\| = \sqrt{3^2+4^2} = 5$

unit vector $= \frac{\langle 3,4\rangle}{5} = \langle \frac{3}{5}, \frac{4}{5}\rangle$

$\vec{v} = 7\langle \frac{3}{5}, \frac{4}{5}\rangle$

$= \langle \frac{21}{5}, \frac{28}{5}\rangle = \boxed{\frac{21}{5}i + \frac{28}{5}j}$

Day 4: Vectors and Dot products

Dot Product

Definition of Dot Product:

The dot product of u = $\langle u_1, u_2 \rangle$ and v = $\langle v_1, v_2 \rangle$ is given by: $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

Properties of the Dot Product:

Let u, v, and w be vectors in the plane or in space and let "c" be a scalar.

(#)

$$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$
$$c(\vec{u} \cdot \vec{v}) = c\vec{u} \cdot \vec{v} = \vec{u} \cdot c\vec{v}$$
$$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$$

Note: the dot product of two vectors is a scalar (a real number), not a vector and can be positive, negative or zero.

Find each dot product.

1. $\langle 4,5 \rangle \cdot \langle 2,3 \rangle$

$= 4(2) + 5(3)$
$= 8 + 15$
$= \boxed{23}$

2. $\langle 2,-1 \rangle \cdot \langle 1,2 \rangle$

$= 2(1) + -1(2)$
$= 2 + -2$
$= \boxed{0}$

3. $\langle 0,3 \rangle \cdot \langle 4,-2 \rangle$

$= 0(4) + 3(-2)$
$= 0 + -6$
$= \boxed{-6}$

Let **u** = $\langle -1,3 \rangle$, **v** = $\langle 2,-4 \rangle$ and **w** = $\langle 1,-2 \rangle$. Find each dot product.

4. $\vec{v} \cdot \vec{w}$

$= 2(1) + -4(-2)$
$= 2 + 8 = \boxed{10}$

5. $\vec{u} \cdot 2\vec{v}$

$= \langle -1,3 \rangle \cdot \langle 4,-8 \rangle$
$= -1(4) + 3(-8)$
$= -4 + -24$
$= \boxed{-28}$

6. $(\vec{u} \cdot \vec{v})\vec{w}$

$= (-1(2) + 3(-4))\vec{w}$
$= (-2 + -12)\vec{w}$
$= -14\vec{w}$
$= \boxed{\langle -14, 28 \rangle}$

7. Let **u** = $\langle -1,3 \rangle$, use the dot product to find the magnitude of **u**.

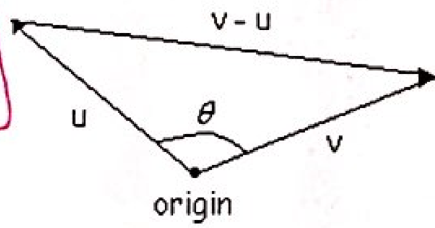
$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$

$\vec{u} \cdot \vec{u} = -1(-1) + 3(3)$
$= 1 + 9$
$= 10$
$\|\vec{u}\|^2 = 10$
$\boxed{\|\vec{u}\| = \sqrt{10}}$

The Angle Between Two Vectors:

The angle between two nonzero vectors is the angle θ $0 \le \theta \le \pi$, between their respective standard position vectors. This angle can be found using the dot product.

If θ is the angle between two nonzero vectors **u** and **v**, then: $\boxed{\cos\theta = \dfrac{\vec{u}\bullet\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}}$



Let **u** = $\langle 4,3\rangle$, **v** = $\langle 3,5\rangle$ and **w** = $\langle -8,-6\rangle$ find the angle between the vectors:

8. **u** and **v**

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{(4)(3)+(3)(5)}{5\,(\sqrt{34})}$$

$\|\vec{u}\| = \sqrt{4^2+3^2} = 5$

$\|\vec{v}\| = \sqrt{3^2+5^2} = \sqrt{34}$

$$= \frac{27}{5\sqrt{34}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{27}{5\sqrt{34}}\right)$$

$$\boxed{\theta = 22.2^\circ}$$

9. **u** and **w**

$\|\vec{u}\| = 5$

$\|\vec{w}\| = \sqrt{(-8)^2+(-6)^2} = 10$

$$\cos\theta = \frac{\vec{u}\cdot\vec{w}}{\|\vec{u}\|\,\|\vec{w}\|}$$

$$= \frac{4(-8)+3(-6)}{5(10)}$$

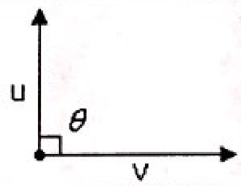
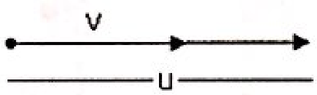
$$= \frac{-50}{50}$$

$\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$$\boxed{\theta = 180^\circ}$$

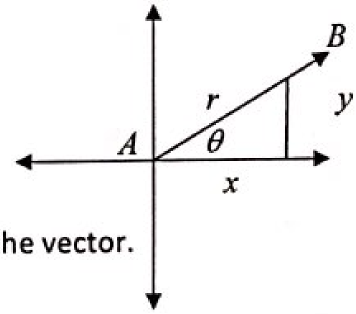
When you use the dot product $\cos\theta = \dfrac{\vec{u}\bullet\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$ to find the angle between two vectors if you get 0, 1, or – 1 your results have special meaning for vectors.

		
$\cos\theta = -1$	$\cos\theta = 0$	$\cos\theta = 1$
Opposite direction = 180°	90° angle	Same direction = 0°
Parallel Vectors	Orthogonal Vectors	Parallel Vectors

<u>Day 5</u>: Vectors and Parametric Equations

Quantities in mathematics and physics often must be measured both in **magnitude** and **direction**. Such quantities are known as **vectors**. We can represent vectors using **directed line segments**.

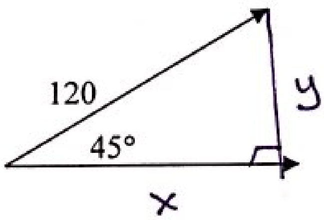
Pictured to the right is vector $\overrightarrow{AB}$.



- A is called the **<u>initial point</u>** of the vector.
- B is called the **<u>terminal point</u>**.
- The length of line segment $\overline{AB}$ represents the **magnitude** of the vector.
- Let r = magnitude of the vector. R is always positive.
- Its direction angle A is the angle it makes with the positive x-axis
- The vector can be broken into components (an x-component and a y-component)

$$x = r\cos\theta \qquad \left(\cos\theta = \frac{x}{r}\right)$$
$$y = r\sin\theta \qquad \left(\sin\theta = \frac{y}{r}\right)$$

1. Find the x and y components for a vector whose magnitude is 120 and direction is 45°.



x-component : $120\cos 45° = 120\,\frac{\sqrt{2}}{2} = \boxed{60\sqrt{2}}$

y-component : $120\sin 45° = 120\,\frac{\sqrt{2}}{2} = \boxed{60\sqrt{2}}$

Parametric Equations

A parametric equation is a set of equations (one for x and one for y) that will allow us to look at 3 variables at once: horizontal distance (x), vertical distance (y), and time (t).

We will use the fact that $\quad Distance = Rate \cdot Time$

x component

Horizontal Distance: (x) $\quad$ **Distance = rate · t** $\quad$ so $\quad x = rcos\theta \cdot t$

y component

(y) **Vertical Distance:** $\quad h = -16t^2 + v_0 t + h_0$ $\quad$ so $\quad y = -16t^2 + rsin\theta \cdot t + h_0$

(Remember the height of the object will be affected by gravity. That is why we are using the position equation above.)

2. If a man hits a golf ball at a 25° angle with an initial force of 210 ft/sec, how far will it be from the ground 5 sec. after it is hit?

θ r $h_0 = 0$

a. Write your parametric equations $x_1 = 210\cos(25°) \cdot t$

$y_1 = -16t^2 + 210\sin(25°) \cdot t$

b. Use one of your equations to determine how far from the ground the ball will be after 5 sec.

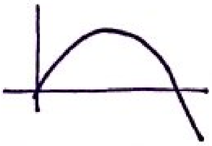
*Vertical distance so use y

$y = -16(5)^2 + 210\sin 25° (5) = \boxed{43.75 \text{ feet}}$

Graphing Parametrics

- Set your calculator to **parametric and degree mode.**
- The y = key will produce $x_{1T} =$ and $y_{1T} =$ for entering equations in terms of t (which can be obtained using the x , t , θ key).
- Your window will determine the interval for t ($T_{\min} - T_{\max}$), how much you want to increase t at a time (T_{step}) and the viewing rectangle ($x_{\min} - x_{\max}$ *by* $y_{\min} - y_{\max}$)

3. Graph the flight path for the golf ball in problem #2 on your graphing calculator. Use the following window: [-10, 1500] x [-200, 200]



* What happens if you increase or decrease tmin & tmax

* What does tstep do?

4. Use your graphing calculator to do the following.

a. Find the height of the ball after 5 seconds. $\boxed{43.75 \text{ feet}}$ Y1(5) = 43.75 or trace

y = ?

What is y when t = 5?

b. Find the horizontal distance of the ball after 5 seconds.

What is x when t = 5? $\boxed{951.62 \text{ feet}}$

c. Approximate the maximum height the ball will reach. (Use the table and be accurate to 2 decimal places.)

Tbl Set
Start @ 3
Δtbl = .1

look @ y – find largest y-value

$\approx \boxed{123.06 \text{ feet}}$

d. Approximate the time (2 decimal places) and horizontal distance (nearest whole number) where the ball will reach the ground. y = 0

Tbl Set
Start @ 5.55
Δtbl = .001
or Start @ 5.5
Δtbl = .01

$\boxed{\begin{array}{l}\text{time} \approx 5.55 \text{ sec} \\ \text{Horiz Distance} = 1056 \text{ feet}\end{array}}$

h_0 θ

5. A man throws a ball into the air from the top of a 200 ft. building at a 50° angle with an initial velocity of 80 ft. per second.

r

a. Write the parametric equations $x_1 = 80\cos(50°)\cdot t$

$y_1 = -16t^2 + 80\sin(50°)t + 200$

b. How far will it be from the ground 2 seconds after he throws it?

y t

$y = -16(2)^2 + 80\sin 50°(2) + 200 = \boxed{258.57 \text{ feet}}$

c. To 2 decimal places, when will it hit the ground?

$t = ?$

$y = 0$

$0 = -16t^2 + 80\sin 50°(t) + 200$

$$\frac{-(80\sin 50°) \pm \sqrt{(80\sin 50°)^2 - 4(-16)(200)}}{2(-16)} = \boxed{5.94 \text{ sec}}$$

Wind and Fences

We will only consider a horizontal wind. This will either increase or decrease your horizontal speed ($r\cos\theta$). So $x = (r\cos\theta \pm wind)\cdot t$

+ if "tail" wind

− if "head" wind

6. A baseball is hit when the ball is 3 ft. above the ground and leaves the bat with an initial velocity of 150 ft/sec and at an angle of elevation of 20°. A 6 mph wind is blowing in a horizontal direction against the batter. A 20 ft. high fence is 400 ft. from home plate. (hint: 60 mph = 88 ft/sec) So ... 6 mph = 8.8 ft/sec make units match

a. Write your parametric equations $x(t) = (150\cos(20°) - 8.8)t + 3$

$y(t) = -16t^2 + 150\sin 20° t + 3$

b. Will the hit go over the fence and be a home run? Explain your reasoning.

X = 400 feet — What is time?

$400 = (150\cos(20°) - 8.8)t$

$\frac{400}{(150\cos 20° - 8.8)} = t = 3.0268$

→ Height when $t = 3.0268$

$y = -16(3.0268)^2 + 150\sin 20°(3.0268) + 3$

$= \boxed{11.67 \text{ ft}}$

* No Home Run :(

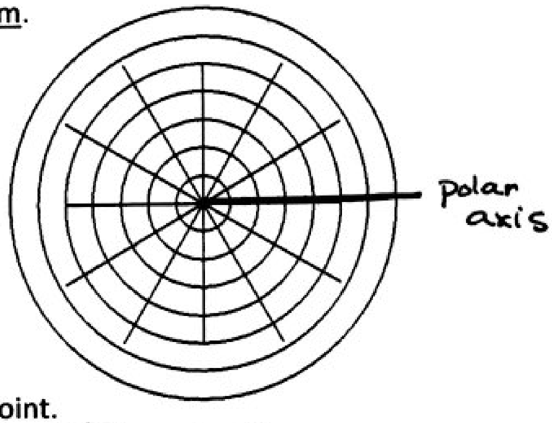
c. What is the maximum height that the ball will reach?

* use table – look for maximum y

$\boxed{\approx 44.12 \text{ feet}}$

Day 6: Polar Coordinates and Equations

There are 2 major coordinate systems in mathematics. The first of these is the rectangular coordinate system. It is the one we have been using so far. While many equations can be graphed from simple equations in the rectangular coordinate system, some more complicated curves can be more simply graphed in a second coordinate system- the polar coordinate system.



The polar coordinate system is formed using:

Pole (or origin) A fixed point

Polar Axis A fixed initial ray

Polar Coordinates $\boxed{(r, \theta)}$ as follows:

- r = directed distance from the origin to the point.
  *r does not need to be positive – this is different from cartesian coordinates
- θ = directed angle from the polar axis to the ray from the origin to the point.

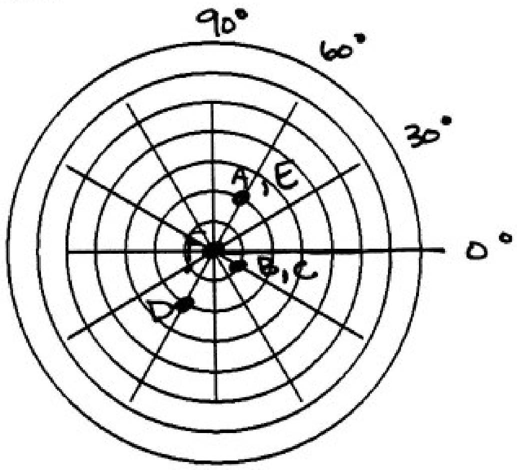
Polar coordinates for a point P are not unique.
For example: (2 , 30°), (2 , 390°), (2 , -330°), and (-2 , 210°) all represent the same point. This is different than rectangular coordinates for a point which are unique.

1. Plot the following points in the coordinate plane:

a. (2 , 60°) b. (1 , -30°)

c. (1 , 330°) d. (-2 , 60°)

e. (-2 , 240°) f. (0 , 90°)



2. Find 3 alternate polar representations for the point (5 , 210°). At least one of your representations must contain a negative r-value.

(5, -150°)
(5, 570°)
(-5, 30°)

Coordinate Conversions and Equation Conversions

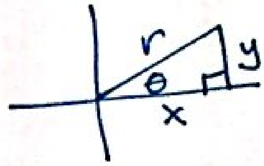
Points and equations can be converted from polar form to rectangular form or vice versa using the following equations or conversion:

* Watch Quadrants

$x = r\cos\theta$

$y = r\sin\theta$

$$\boxed{\begin{aligned}&\tan\theta = \frac{y}{x} \quad (x \neq 0)\\ &r^2 = x^2 + y^2 \quad or \quad r = \pm\sqrt{x^2+y^2}\end{aligned}}$$



3. Convert $\left(\underset{r}{\sqrt{3}}, \underset{\theta}{\frac{\pi}{6}}\right)$ from polar to rectangular form.

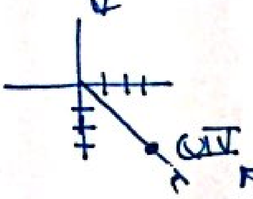
find x & y

$x = \sqrt{3}\cos \pi/6 = \sqrt{3}\left(\sqrt{3}/2\right) = 3/2$

$y = \sqrt{3}\sin \pi/6 = \sqrt{3}\left(1/2\right) = \sqrt{3}/2$

$\boxed{\left(3/2, \sqrt{3}/2\right)}$

4. Convert (3 , -3) from rectangular to polar form. Express using a positive r-value and then try it with a negative r-value.



$r = \pm\sqrt{(3)^2+(-3)^2} = \pm\sqrt{18} = \pm 3\sqrt{2}$

$\tan\theta = \frac{-3}{3}$

$\tan\theta = -1$

$\theta = 315°$

$\boxed{(3\sqrt{2}, 315°)}$

OR $(-3, 135°)$ or $(3, -45°)$ ....

5. Convert from polar form and isolate r:

a. $x = 4$

$r\cos\theta = 4$

$r = \frac{4}{\cos\theta}$

$\boxed{r = 4\sec\theta}$

b. $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$\frac{\cancel{r}\sin\theta}{\cancel{r}\cos^2\theta} = \frac{r^2\cos^2\theta}{r\cos^2\theta}$

$\frac{\sin\theta}{\cos^2\theta} = r$

$\boxed{r = \sec\theta\tan\theta}$

6. Convert to rectangular form:

a. $r = 2$

$\sqrt{x^2+y^2} = 2$

$\boxed{x^2+y^2 = 4}$

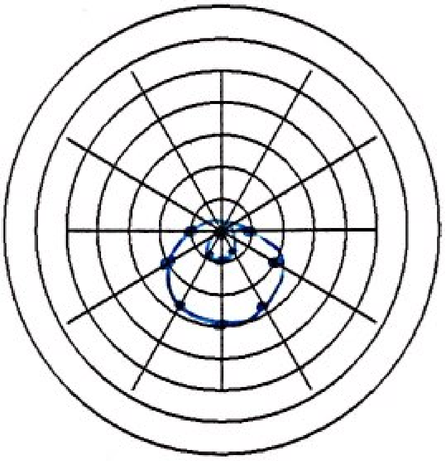
b. $r \cdot r = 2\sin\theta \cdot r$

$r^2 = 2r\sin\theta$

$\boxed{x^2+y^2 = 2y}$

7. Sketch the graph of the polar equation $\boxed{r = 1 - 2\sin\theta}$ by filling in the table below, plotting the points, and connecting them.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
r	1	0	-.732	-1	-.732	0	1	2	2.73	3	2.73	2	1



Now confirm your graph in the following way:

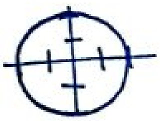
- Set your calculator in polar mode, degree mode, and use the following window:

$$\theta\text{min} = 0°\ ,\ \theta\text{max} = 360°,\ \theta\text{step} = 10°, -6 \le x \le 6, -4 \le y \le 4$$

8. Graph the following polar equations on your calculator:

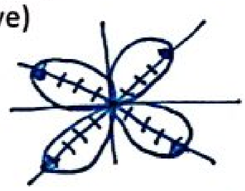
a. $r = 2$ What do you think the graph will look like?

radius=2....

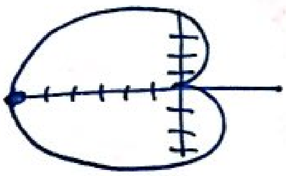


b. $r = 4\sin 2\theta$ (rose curve)

petal length



c. $r = 3 - 3\cos\theta$ (cardiod)

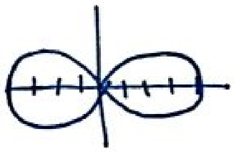


d. $r^2 = 16\cos 2\theta$

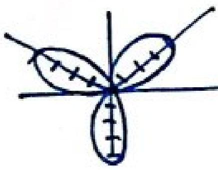
$r = \sqrt{16\cos 2\theta}$

$r = -\sqrt{16\cos 2\theta}$

*Change θstep to 5.



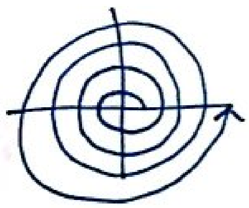
e. $r = 4\sin 3\theta$ (rose curve)



f. $r = \theta + 360$ (spiral of Archimedes)

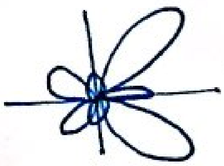
* Need to extend θ_{max}

* use zoomfit



g. $r = e^{\cos\theta} - 2\cos(4\theta) + \sin^5\left(\frac{\theta}{12}\right)$

* use window given above w/ $\theta_{step} = 5$



Butterfly :)