Teacher Copy

Unit 3 Notes / Secondary 3 Honors

Day 1: **Factoring & Polynomial Division**

Factoring Methods: **GCF, Backwards Foil, Difference of Squares, Sum/Difference of Cubes, Grouping**

Formulas:

$a^2-b^2=(a+b)(a-b)$ — diff of □

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

} sum / diff of cubes

1. Factor each of the polynomials:

a. $3x^4+2x^2-8$

$\boxed{(3x^2-4)(x^2+2)}$ +/− 1,8 2,4

b. $25x^2-1$

$\boxed{(5x+1)(5x-1)}$

c. x^2-5x+6

$\boxed{(x-2)(x-3)}$ 1,6 2,3

d. $2x^4+10x^3+12x^2$

$= 2x^2(x^2+5x+6)$ 2,3 1,6

$= \boxed{2x^2(x+2)(x+3)}$

e. $x^3-2x^2-13x+26$

$= x^2(x-2)-13(x-2)$

$= \boxed{(x-2)(x^2-13)}$

f. x^3-64

$= \boxed{(x-4)(x^2+4x+16)}$

Polynomial Long Division:

- **Can be used to factor a polynomial when your factoring methods don't work.** — Always try factoring "methods" before using division
- **You must represent the divisor in descending order of exponents and use zeros for placeholders.** — if any missing powers of x
- **Any polynomial can be divided using long division**

2. Divide $x^3-2x^2-13x+26$ by $x-2$ using long division.

$$\begin{array}{r}
x^2 \quad\quad -13 \\
x-2\ \overline{)\ x^3-2x^2-13x+26} \\
-x^3+2x^2 \quad\quad\quad\quad \\
\hline
0 -13x+26 \\
+13x-26 \\
\hline
0
\end{array}$$

$\boxed{x^2-13}$

✱ Since no remainder, $(x-2)$ is a factor of $x^3-2x^2-13x+26$

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Divide using long division:

3. $(x^3-1) \div (x-1)$

$$
\begin{array}{r}
x^2 + x + 1 \\
x-1 \,\big)\, \overline{\cancel{x^3} + 0x^2 + 0x - 1} \\
-\cancel{x^3} + x^2 \\
\hline
\cancel{x^2} + 0x \\
-\cancel{x^2} + x \\
\hline
\cancel{x} - 1 \\
-\cancel{x} + 1 \\
\hline
0
\end{array}
$$

$\boxed{x^2+x+1}$

4. $(2x^4+4x^3-5x^2+3x-2) \div (x^2+2x-3)$

$$
\begin{array}{r}
2x^2 + 1 \\
x^2+2x-3 \,\big)\, \overline{\cancel{2x^4} + \cancel{4x^3} - 5x^2 + 3x - 2} \\
-\cancel{2x^4} + \cancel{4x^3} + 6x^2 \\
\hline
\cancel{x^2} + 3x - 2 \\
-\cancel{x^2} + 2x + 3 \\
\hline
x + 1
\end{array}
$$

$\boxed{2x^2+1 + \frac{x+1}{x^2+2x-3}}$

Synthetic Division:

- Synthetic Division uses the ROOT form as the divisor.
- Synthetic Division can only be used when the divisor is LINEAR (x to the power of one)
- ** You must represent the dividend in descending order of exponents and use zeros for placeholders.
- Your answer must be interpreted back into a solution using variables
- It is tricky to divide with a divisor when the coefficient of x is not equal to one.
  → I would use long division for these

5. Divide $x^3-2x^2-13x+26$ by $(x-2)$ using synthetic division.

root is 2.

$$
\begin{array}{r|rrrr}
2 & 1 & -2 & -13 & 26 \\
 & & 2 & 0 & -26 \\
\hline
 & 1 & 0 & -13 & 0
\end{array}
$$

$\boxed{x^2-13}$

6. Divide using synthetic division.

a. $(x^4-10x^2-2x+4) \div (x+3)$

root is -3

$$
\begin{array}{r|rrrrr}
-3 & 1 & 0 & -10 & -2 & 4 \\
 & & -3 & 9 & 3 & -3 \\
\hline
 & 1 & -3 & -1 & 1 & 1
\end{array}
$$

$\boxed{x^3-3x^2-x+1 + \frac{1}{x+3}}$

b. $(8x^3-1) \div (x-1)$

$$
\begin{array}{r|rrrr}
1 & 8 & 0 & 0 & -1 \\
 & & 8 & 8 & 8 \\
\hline
 & 8 & 8 & 8 & 7
\end{array}
$$

$\boxed{8x^2+8x+8 + \frac{7}{x-1}}$

Day 2: Zeros of Polynomials/ Factor, Remainder & Rational Root Theorems

1. Factor the following polynomial. $f(x)=x^2-2x-15$

$= (x-5)(x+3$

$f(-3) = (-3)^2-2(-3)-15=\boxed{0}$ $f(5) = \boxed{0}$

Factor Theorem: If $(x-a)$ is a factor of a polynomial $f(x)$ then $f(a)=0$.

2. Determine if $(x-4)$ is a factor of $f(x)=x^3-10x^2+37x-52$.

$f(4)=(4)^3-10(4)^2+37(4)-52$
$=\boxed{0}$ $\boxed{\text{Yes } (x-4) \text{ is a factor}}$

3. Divide the function $f(x)$ by $(x+1)$. What is your remainder? What is $f(-1)$?

-1	1	-10	37	-52
		-1	11	-48
	1	-11	48	-100

$f(-1)=(-1)^3-10(-1)^2+37(-1)-52$
$=-100$

Remainder $=-100$

Remainder Theorem:
If a polynomial $f(x)$ is divided by $(x-a)$ then $f(a)=R$, where R is the remainder.

4. What is the remainder if $f(x)=3x^3+8x^2+5x-7$ is divided by $(x+2)$? -2

$f(-2)=3(-2)^3+8(-2)^2+5(-2)-7$
$=\boxed{-9}$

5. Is 3 a root of $3x^3-9x^2-8x+24$?

$f(3)=3(3)^3-9(3)^2-8(3)+24$
$=0$ $\boxed{\text{Yes, 3 is a root}}$

6. Find all the zeros of $f(x)=x^3-10x^2+37x-52$. (Hint: See problem #2) → one of the factors is $x-4$ ☺

4	1	-10	37	-52
		4	-24	52
	1	-6	13	☺✓

$f(x)=(x-4)(x^2-6x+13)$

$x-4=0$ $x^2-6x+13=0$

$\boxed{x=4}$

$x=\frac{6\pm\sqrt{36-4(1)(13)}}{2(1)}$

$=\frac{6\pm\sqrt{-16}}{2}$

$=\frac{6\pm 4i}{2}=\boxed{3\pm 2i}$

Zeros
4, $3\pm 2i$

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Rational Zero Test:

- Gives you a ***possible*** list of all rational roots (not imaginary or irrational) of a function.
- **Possible Rational Roots:** $\pm\frac{factors\ of\ the\ constant}{factors\ of\ the\ leading\ coefficient}$
- Once you make your list, use a trial-and-error method or Synthetic Division and the Remainder Theorem to determine which, if any, are actual zeros of the polynomial.

7. Find the zeros of $x^4+x^3-7x^2-x+6=0$ constant $=6$, Lead. Coeff. $=1$

Step 1: Make a list of the *possible* Rational Roots.

$\pm\frac{1,2,3,6}{1} = \boxed{\pm1, \pm2, \pm3, \pm6}$

Step 2: Determine one of the zeros using Remainder Theorem or graph of the polynomial.

$f(1)=0$ ✓

So 1 is a zero

Step 3: Use synthetic division to help you write the polynomial in factored form.

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -7 & -1 & 6 \\ & & 1 & 2 & -5 & -6 \\ \hline & 1 & 2 & -5 & -6 & 0 \end{array}$$

☺✓

$f(x)=(x-1)(x^3+2x^2-5x-6)$

need to factor this more

Step 4: Repeat steps 2 & 3 for the cubic expression.

Find another zero: $f(-1)=0$ ✓

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

☺✓

$f(x)=(x-1)(x+1)(x^2+x-6)$

Step 5: Factor or use the quadratic formula to find the final two factors or roots of the remaining quadrati

$0=(x-1)(x+1)(x^2+x-6)$

$x-1=0$ $\quad x+1=0$ $\quad x^2+x-6=0$

$x=1$ $\quad x=-1$ $\quad (x+3)(x-2)=0$

$x=-3$ $\quad x=2$

Zeros

1, -1, -3, 2

8. Find all of the zeros for $h(x) = 2x^4 + 4x^3 - 10x^2 - 8x + 12$. Write the function in completely factored form.

List

$\pm \dfrac{1,2,3,4,6,12}{1,2}$

$\pm 1, \pm 1/2, \pm 3, \pm 3/2, \pm 4, \pm 6, \pm 12$

Find one that works:

$h(1) = \underline{\underline{0}}$ ✓

$$\begin{array}{r|rrrrr} 1 & 2 & 4 & -10 & -8 & 12 \\ & & 2 & 6 & -4 & -12 \\ \hline & 2 & 6 & -4 & -12 & ☺ \end{array}$$ ✓

Factor Polynomial

$h(x) = (x-1)(2x^3+6x^2-4x-12)$

$= (x-1)(2x^2(x+3)-4(x+3))$

$= (x-1)(x+3)(2x^2-4)$

$= 2(x-1)(x+3)(x^2-2)$

Solve factors to find zeros.

$x-1=0 \quad x+3=0 \quad x^2-2=0$

$x=1 \quad x=-3 \quad x^2=2$

$x = \pm\sqrt{2}$

$\boxed{\text{Zeros: } 1, -3, \pm\sqrt{2}}$

9. Find all of the zeros of $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$.

List

$\pm 1, \pm 2, \pm 4, \pm 8$

Find one that works

$f(1)=0$ ✓

$f(-2)=0$ ✓

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 1 & 2 & -12 & 8 \\ & & 1 & 1 & 2 & 4 & -8 \\ \hline & 1 & 1 & 2 & 4 & -8 & ☺ \end{array}$$

$$\begin{array}{r|rrrrr} -2 & 1 & 1 & 2 & 4 & -8 \\ & & -2 & 2 & -8 & 8 \\ \hline & 1 & -1 & 4 & -4 & ☺ \end{array}$$

Factor Polynomial

$f(x) = (x-1)(x^4+x^3+2x^2+4x-8)$

$= (x-1)(x+2)(x^3-x^2+4x-4$

$= (x-1)(x+2)(x^2(x-1)+4(x-$

$= (x-1)(x+2)(x-1)(x^2+4)$

Solve factors to find zeros

$x-1=0 \quad x+2=0 \quad x-1=0 \quad x^2+4=0$

$x=1 \quad x=-2 \quad x=1 \quad x^2=-4$

$x = \pm\sqrt{-4} = \pm 2i$

$\boxed{\text{Zeros: } 1, -2, 1, \pm 2i}$

Day 3: **Polynomial Inequalities**

Linear Inequalities: **Just isolate x.**

1. Find the solution set for $5x-7>3x+9$.

$-3x^{+7} \quad -3x^{+7}$

$2x>16$

$\boxed{x>8}$

2. Solve $-3 \le 6x-1<3$ and graph the solution set on a number line.

$+1 \quad +1 \quad +1$

$\dfrac{-2}{6} \le \dfrac{6x}{6} < \dfrac{4}{6}$

$\boxed{-\frac{1}{3} \le x < \frac{2}{3}}$

$-\frac{1}{3} \qquad \frac{2}{3}$

Quadratic & Higher order Polynomial Inequalities:

- **Use a graph or number line**

3. Solve algebraically… use a number line. $2x^4-4x^2>0$ → means "+"

$2x^4-4x^2=0$

$2x^2(x^2-2)=0$

$x=0, \pm\sqrt{2}$

- Find zeros – mark on # line
- use test points to determine where function is + or –

Test points:

$-2: 2(-2)^4-4(-2)^2=16$

$-1: 2(-1)^4-4(-1)^2=-2$

$1: 2(1)^4-4(1)^2=-2$

$2: 2(2)^4-4(2)^2=16$

$+ \quad - \quad - \quad +$

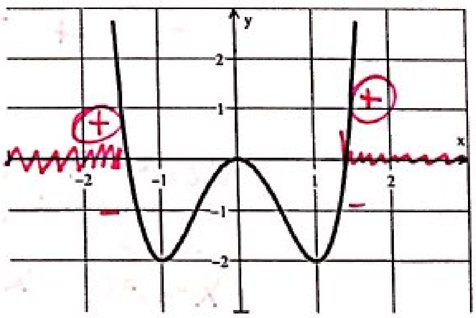
$-\sqrt{2} \qquad 0 \qquad \sqrt{2}$

Solution

$x<-\sqrt{2} \quad x>\sqrt{2}$

4. Given the graph of $f(x)=2x^4-4x^2$, find the intervals where $f(x)>0$.

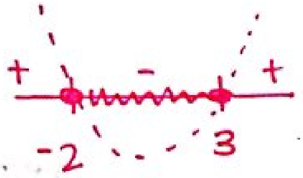
- Mark zeros on # line
- use graph to decide where f(x) is + or –



$-\sqrt{2} \qquad 0 \qquad \sqrt{2}$

$\boxed{x<-\sqrt{2} \qquad x>\sqrt{2}}$

5. Solve for x: $(x-3)(x+2) \le 0$



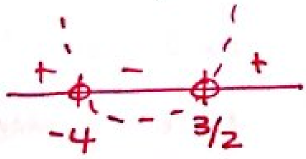
$\boxed{-2 \le x \le 3}$

6. Solve for x: $2x^2 + 5x > 12$

$2x^2 + 5x - 12 > 0$

$(2x-3)(x+4) > 0$

$x = 3/2, -4$



$\boxed{\begin{array}{l} x < -4 \\ x > 3/2 \end{array}}$

7. Solve for x: $6x^3 - 21x^2 - 12x \ge 0$

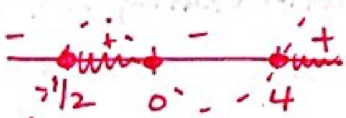
EB

L↓ R↑

$3x(2x^2 - 7x - 4) = 0$

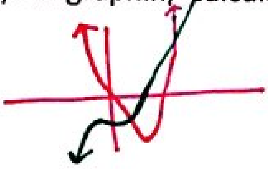
$3x(2x+1)(x-4) = 0$

$x = 0, -1/2, 4$



$\boxed{\begin{array}{l} -\frac{1}{2} \le x \le 0 \\ x \ge 4 \end{array}}$

8. Use your graphing calculator to solve for x: $x^4 - 5x > x^3 + 2x^2 - 4$



$\boxed{(-\infty, .625) \quad (2.407, \infty)}$

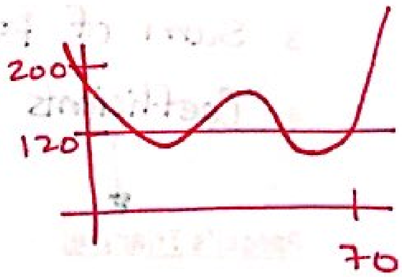
9. The average blood sugar (also known as glucose) levels in a person's blood should be between 70 and 100 mg/dL (milligrams per deciliter) one hour after eating. A person with Type 2 diabetes strives to keep glucose levels under 120 mg/dL with diet and exercise in order to avoid insulin injections. Glucose levels of one individual over the span of 72 hours can be represented with the polynomial function,

$$b(t) = 0.000139x^4 - 0.0188x^3 + 0.8379x^2 - 13.55x + 176.51$$

where glucose levels is a function of the number of hours.

a. For what hours were the glucose levels greater than 120 mg/dL?

from: 0 to 6.28 hrs
23.9 to 45.2 hrs
59.5 to 72 hrs



b. For what hours were the glucose levels less than 120 mg/dL?

from 6.28 to 23.9 hrs
45.2 to 59.5 hrs

Day 4: Binomial Theorem and Pascal's Triangle

Needed Skills:
Recall that ! is called a *factorial* symbol. Ex: $4!=4\times3\times2\times1$
It will be necessary to make a rule that $0!$ always equals 1. Ex: $0!=1$

Combinations: ${}_nC_r=\binom{n}{r}=\frac{n!}{(n-r)!r!}$ → (nCr in Math/Prob)

Find the following by hand. Check some of them by calculator:

1. ${}_8C_2$ $=\frac{8!}{6!2!}=\boxed{28}$
2. $\binom{10}{3}=\frac{10!}{7!3!}=\boxed{120}$
3. ${}_7C_0=\frac{7!}{7!0!}=\boxed{1}$
4. $\binom{8}{8}=\frac{8!}{0!8!}=\boxed{1}$
5. ${}_{250}C_2$ $\frac{250!}{248!2!}=\boxed{31125}$

Binomial = Polynomial with exactly 2 terms

Look at the expansion of $(x+y)^n$ for several values of n.

$(x+y)^0=1$
$(x+y)^1=x+y$
$(x+y)^2=x^2+2xy+y^2$
$(x+y)^3=x^3+3x^2y+3xy^2+y^3$
$(x+y)^4=x^4+4x^3y+6x^2y^2+4xy^3+y^4$
$(x+y)^5=x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

There are several observations we can make about these expansions.

1. Each expansion includes (n+1) terms
2. Powers of x decrease, Powers of y increase
3. Sum of powers in each term is n
4. Coefficients increase then decrease symmetrically.

↓

Pascal's Triangle:

* Find the pattern and fill in next 2 rows :)

$n=0$							1							
$n=1$						1		1						
$n=2$					1		2		1					
$n=3$				1		3		3		1				
$n=4$			1		4		6		4		1			
$n=5$		1		5		10		10		5		1		
$n=6$	1		6		15		20		15		6		1	

The Binomial Theorem:

$(x+y)^n = x^n + nx^{n-1}y + {}_nC_r x^{n-2}y^2 + \ldots + \boxed{{}_nC_r x^{n-r} y^r} + \ldots + nxy^{n-1} + y^n$

Can use pascal's Δ or $_nC_r$ for coeff. (r = power y is raised to)

Write the expansion of the following expressions.

6. $(x+1)^3$

$= 1x^3(1)^0 + 3x^2 1^1 + 3x^1 1^2 + 1x^0 1^3$

$= \boxed{x^3 + 3x^2 + 3x + 1}$

7. $(2x-3)^4$

1 4 6 4 1

$= 1(2x)^4(-3)^0 + 4(2x)^3(-3)^1 + 6(2x)^2(-3)^2 + 4(2x)^1(-3)^3 + 1(2x)^0(-3)^4$

$= \boxed{16x^4 - 96x^3 + 216x^2 - 216x + 81}$

8. $(x-2y)^4$

1 4 6 4 1

$= (x)^4 + 4(x)^3(-2y)^1 + 6(x)^2(-2y)^2 + 4(x)^1(-2y)^3 + (-2y)^4$

$= \boxed{x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^2}$

9. $(x^2+4)^3$

1 3 3 1

$= (x^2)^3 + 3(x^2)^2(4)^1 + 3(x^2)(4)^2 + 4^3$

$= \boxed{x^6 + 12x^4 + 48x^2 + 64}$

10. Find the 6th term of $(a+2b)^8$.

${}_8C_5(a)^3(2b)^5$

$= 56a^3(32b^5)$

$= \boxed{1792a^3b^5}$

11. Find the coefficient of the term a^6b^5 in the expansion of $(2a-5b)^{11}$.

${}_{11}C_5(2a)^6(-5b)^5$

$= 462(2)^6a^6(-5)^5b^5$

Coeff $= 462(2^6)(-5)^5 = \boxed{-92,400,000}$