Teacher Copy

Unit 7 Notes / Secondary 3 Honors

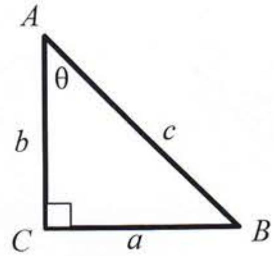
Day 1: Right Triangle Trigonometry

Trigonometry = triangle measurement

Trig Functions:

In the triangle pictured below, angle C is a right angle, while angle A and B are acute angles.
We define the 6 trigonometric functions of θ as follows: ($0 < \theta < 90°$)

Basic	**Reciprocal**
$sin\theta = \frac{Opp}{Hyp}$	$csc\theta = \frac{Hyp}{Opp}$
$cos\theta = \frac{Adj}{Hyp}$	$sec\theta = \frac{Hyp}{Adj}$
$tan\theta = \frac{Opp}{Adj}$	$cot\theta = \frac{Adj}{Opp}$



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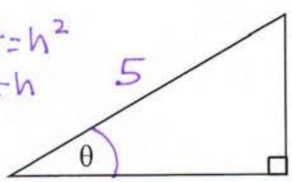
Pythagorean Theorem: $a^2 + b^2 = c^2$

Examples: Find the six trigonometric values for the triangle(s) shown below.

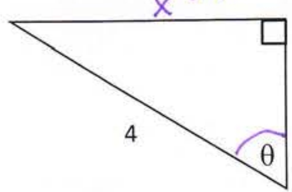
1.

$3^2 + 4^2 = h^2$
$5 = h$



$\sin\theta = \frac{3}{5}$
$\cos\theta = \frac{4}{5}$
$\tan\theta = \frac{3}{4}$
$\csc\theta = 5/3$
$\sec\theta = 5/4$
$\cot\theta = 4/3$

2.

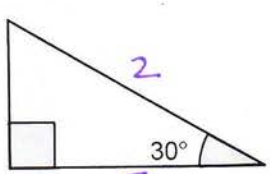
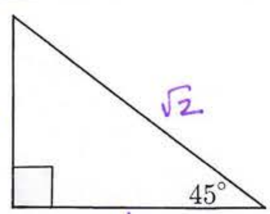


$x^2 + 3^2 = 4^2$
$x^2 = 7$
$x = \sqrt{7}$

$\sin\theta = \frac{\sqrt{7}}{4}$
$\cos\theta = \frac{3}{4}$
$\tan\theta = \frac{\sqrt{7}}{3}$
$\csc\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$
$\sec\theta = \frac{4}{3}$
$\cot\theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

"Special" right triangles:

short leg = s
long leg = $s\sqrt{3}$
hyp = 2s

30-60-90	**45-45-90**
	

legs = s
hyp = $s\sqrt{2}$

Find the following using the special triangles.

3. $\sin 30° = 1/2$
   $\cos 30° = \sqrt{3}/2$
   $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

4. $\sin 60° = \sqrt{3}/2$
   $\cos 60° = 1/2$
   $\tan 60° = \sqrt{3}$

5. $\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   $\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   $\tan 45° = 1$

NOTE: Memorize these values or be able to draw the triangles to get them!!!

Evaluating Trigonometric Functions with a Calculator

- Make sure your calculator is set in the **correct mode** (degrees or radians).
- Sin, cos, and tan ratios can be found directly.
- To find a csc, sec, or cot ratio, you will need to use division.

Finding the ratio if you have been given the angle:

6. Find the following with your calculator to 4 decimal places:

a. $\sin 27^\circ$ $\boxed{.4540}$

b. $\tan 27^\circ$ $\boxed{.5095}$

c. $\sec 27^\circ$ $= \frac{1}{\cos 27^\circ} = \boxed{1.1223}$

d. $\cot 27^\circ$ $= \frac{1}{\tan 27^\circ} = \boxed{1.9626}$

Finding the angle if you have been given the ratio:

Use the **2nd function key (ex.** $\cos^{-1}$**)**

7. Find θ, where $0^\circ < \theta < 90^\circ$, to 4 decimal places:

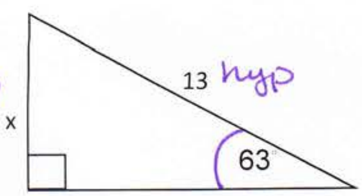
a. $\cos\theta = .293$ $\theta = \cos^{-1}(.293) = \boxed{72.9624^\circ}$

b. $\tan\theta = 1.32$ $= \boxed{52.8533^\circ}$

c. $\sin\theta = .43$ $= \boxed{25.4676^\circ}$

Examples: Solve for the indicated side or angle.

8. Solve for x.

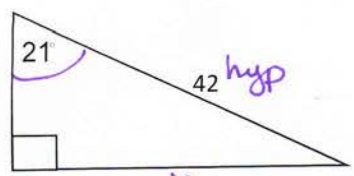


$\sin 63^\circ = \frac{x}{13}$

$13\sin 63^\circ = x$

$\boxed{11.58 = x}$

9. Solve for x.

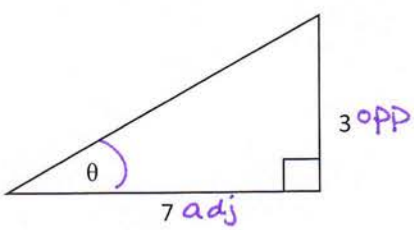


$\cos 21^\circ = \frac{x}{42}$

$42\cos 21^\circ = x$

$\boxed{39.21 = x}$

10. Solve for θ.

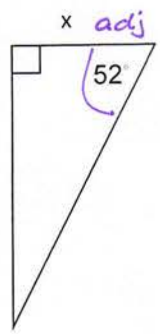


$\tan\theta = \frac{3}{7}$

$\theta = \tan^{-1}\left(\frac{3}{7}\right)$

$\boxed{\theta = 23.2^\circ}$

11. Solve for x.



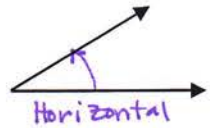
$\tan 52^\circ = \frac{5}{x}$

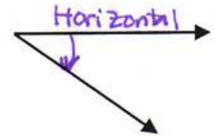
$x(\tan 52^\circ) = 5$

$x = \frac{5}{\tan 52^\circ}$

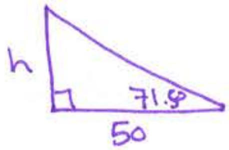
$\boxed{x = 3.906}$

Applications Involving Right Triangles

Angle of elevation: 

Angle of depression: 

12. A surveyor is standing 50 ft. from the base of a large tree. The surveyor measures the angle of elevation to the top of the tree as 71.5°. How tall is the tree?

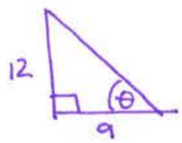


$\tan 71.5° = \frac{h}{50}$

$50 \tan 71.5° = h$

$\boxed{h = 149.43 \text{ ft.}}$

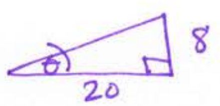
13. A 12-meter flagpole casts a 9-meter shadow. Find θ, the angle of elevation of the sun.



$\tan\theta = \frac{12}{9}$

$\theta = \tan^{-1}\left(\frac{12}{9}\right) = \boxed{53.13°}$

14. A monster truck drives off a ramp in order to jump onto a row of cars. The ramp has a height of 8 feet and a horizontal length of 20 feet. What is the angle θ of the ramp?



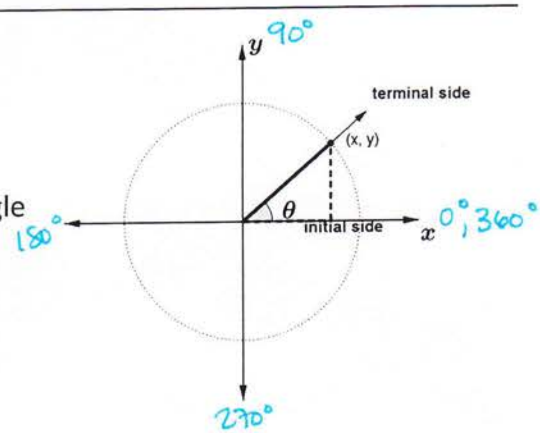
$\tan\theta = \frac{8}{20}$

$\theta = \tan^{-1}(8/20) = \boxed{21.8°}$

Day 2: Trig Functions at any Angle

Standard Position Angles

Let the origin of a coordinate plane be the **vertex** of an **angle** whose **initial side** is the positive x-axis and whose **terminal side** forms an angle measuring θ with respect to the initial side.



Positive Angles = Counterclockwise rotation

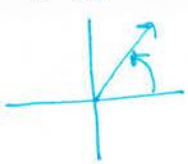
Negative Angles = clockwise rotation

Coterminal angles = angles with the same terminal side, but different measures.

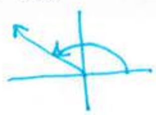
Examples:

Draw the following angles in standard position.

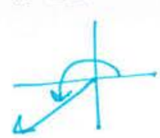
1. 60°



2. 135°



3. 210°



4. 475°

5. -140°

Find one positive and one negative angle that is coterminal with the given angle.

6. 120^0

$120° + 360° = \boxed{480°}$ +

$120° - 360° = \boxed{-240°}$ −

7. -420^0

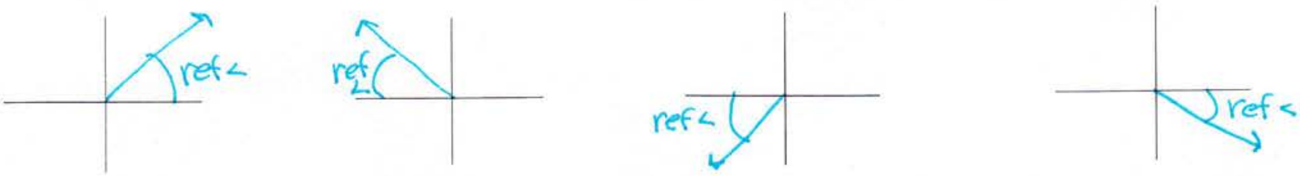
$-420° + 360° = \boxed{-60°}$ −

$-60° + 360° = \boxed{300°}$ +

Reference Angle:

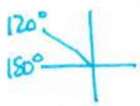
- The **acute** angle between the terminal side and the **x-axis**.
- Always positive.

Q1 Q2 Q3 Q4



Example: Find the measure of the reference angle for the given angles.

8. 120^0



$\boxed{60°}$

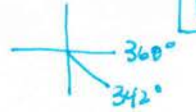
9. 30^0

$\boxed{30°}$

10. 245^0

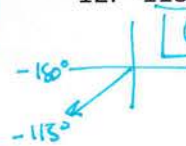
$\boxed{65°}$

11. 342^0



$\boxed{18°}$

12. -115^0

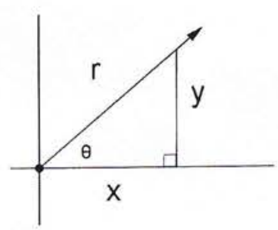


$\boxed{65°}$

Reference Triangle:

- Right triangle containing the reference angle and the x-axis.
- Used to find trig ratios for angles in standard position.

Let θ be an angle in standard position with (x , y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$ as shown in the figure. Then:



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

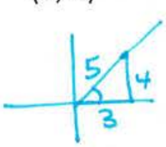
Examples:

Find sin, cos, and tan if the terminal side of θ goes through the given point.

13. (3, 4)

$3^2 + 4^2 = r^2$

$5 = r$

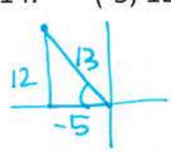


$$\boxed{\begin{array}{l} \sin\theta = \frac{4}{5} \\ \cos\theta = \frac{3}{5} \\ \tan\theta = \frac{4}{3} \end{array}}$$

14. (-5, 12)

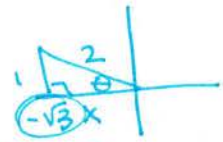
$(-5)^2 + 12^2 = r^2$

$13 = r$



$$\boxed{\begin{array}{l} \sin\theta = \frac{12}{13} \\ \cos\theta = \frac{-5}{13} \\ \tan\theta = -12/5 \end{array}}$$

15. Find $\cos\theta$ if $\sin\theta = \frac{1}{2}$ and θ is in Quadrant II.

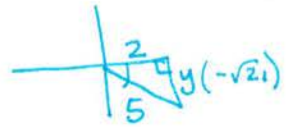


$\boxed{\cos\theta = -\frac{\sqrt{3}}{2}}$

$x^2 + 1^2 = 2^2$

$x = -\sqrt{3}$

16. Find $\tan\theta$ if $\cos\theta = \frac{2}{5}$ and θ is in Quadrant IV.



$2^2 + y^2 = 5^2$

$y^2 = 21$

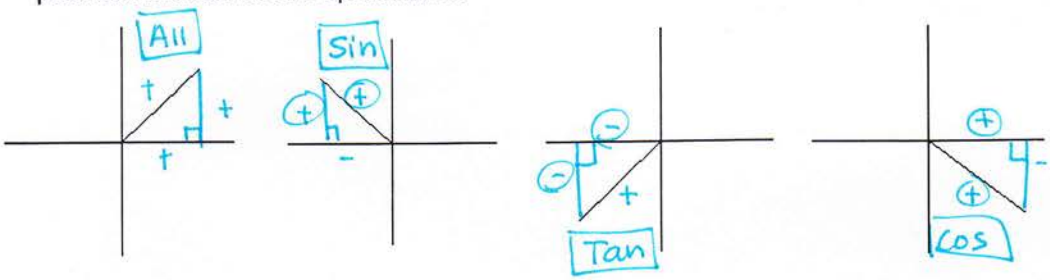
$y = -\sqrt{21}$

$\boxed{\tan\theta = -\frac{\sqrt{21}}{2}}$

Trig Ratios: Find the following ratios using the "special" triangles. These need to be MEMORIZED.

	30°	45°	60°
sin θ	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos θ	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan θ	$\sqrt{3}/3$	1	$\sqrt{3}$

Quadrants: Label the sides of each triangle as "+" or "-". Then identify which of the basic trig ratios would be positive in each of the quadrants.



MEMORIZE!!!

(All Students Take Calc :))

Examples: Evaluate the following **without** using a calculator.
(HINT: use quadrant and reference angle)

17. $\sin 240° = \boxed{-\sqrt{3}/2}$

Q: III sin is (−)

ref∠: 60° sin is $\sqrt{3}/2$

18. $\tan 210° = \boxed{\frac{\sqrt{3}}{3}}$

Q: III

ref∠: 30°

19. $\cos 135° = \boxed{-\frac{\sqrt{2}}{2}}$

Q: II −

ref∠ = 45°

20. $\sec 210° = -\frac{2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$

Q: III

ref∠: 30°

$\cos 210° = -\frac{\sqrt{3}}{2}$

21. $\cot(-60°) = \boxed{-\sqrt{3}/3}$

Q: IV

ref∠: 60°

$\tan(-60°) = -\sqrt{3}$

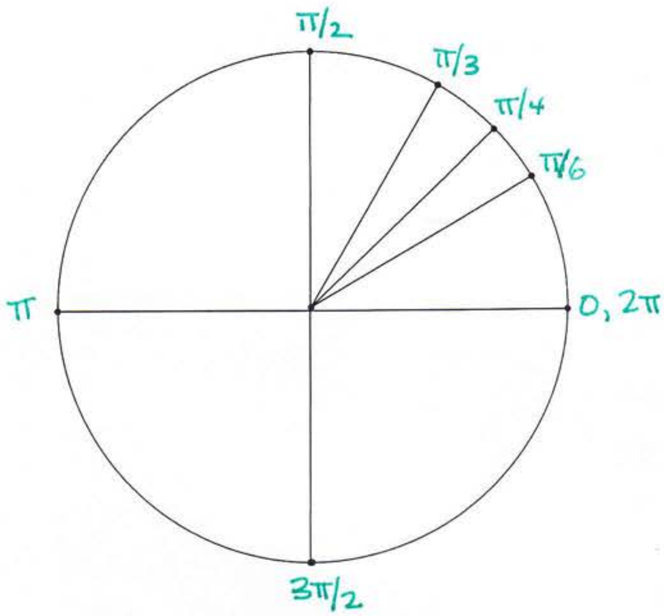
Day 3: Radians

The **unit circle** is a circle with a radius of one unit and a center point at the origin.

Radians:

length of this arc is the radian measure

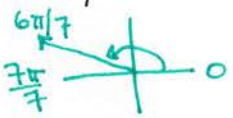
- Different way of measuring angles.
- The length of the arc of the unit circle that is inside the angle you are measuring.



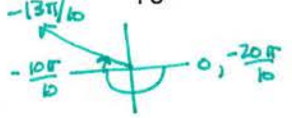
- What is the circumference of the unit circle?
  $C = 2\pi r$, so $C = 2\pi(1) = \boxed{2\pi}$
- So, $\boxed{360° = 2\pi \text{ radians}}$
- What is the distance halfway around? π
- So, $\boxed{180° = \pi}$
- $90° = \pi/2$
- $270° = 3\pi/2$
- $30° = \frac{\pi}{6}$ $\left(\frac{180°}{6} = 30°\right)$
- $45° = \frac{\pi}{4}$ $\left(\frac{180°}{4} = 45°\right)$
- $60° = \frac{\pi}{3}$ $\left(\frac{180°}{3} = 60°\right)$

Graph the following angles in standard position.

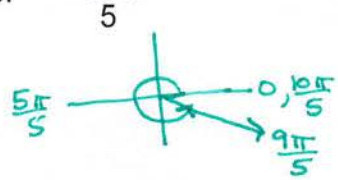
1. $\frac{6\pi}{7}$



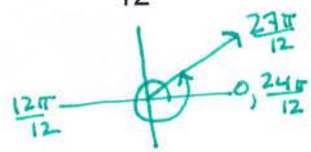
2. $-\frac{13\pi}{10}$



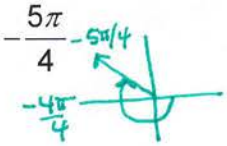
3. $\frac{9\pi}{5}$



4. $\frac{27\pi}{12}$

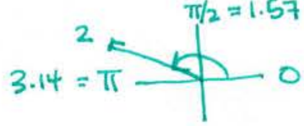


5. $-\frac{5\pi}{4}$



6. -3π

7. 2



Find one positive angle and one negative angle that are coterminal to the given angle. (+/- full rotation)

8. $\frac{\pi}{3}$

$\frac{\pi}{3} + 2\pi = \frac{\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{7\pi}{3}}$ +

$\frac{\pi}{3} - 2\pi = \frac{\pi}{3} - \frac{6\pi}{3} = \boxed{\frac{-5\pi}{3}}$ -

9. $-\frac{10\pi}{7}$

$-\frac{10\pi}{7} + \frac{14\pi}{7} = \boxed{\frac{4\pi}{7}}$ +

$-\frac{10\pi}{7} - \frac{14\pi}{7} = \boxed{\frac{-24\pi}{7}}$ -

Converting Units:

To convert radians to degrees, multiply by $\frac{180°}{\pi}$. → this is equivalent to 1

10. Convert each radian angle to degrees. On d & e, round to 2 decimal places.

a. $\frac{13\pi}{6}$ b. $\frac{3\pi}{4}$ c. $\frac{-2\pi}{3}$ d. 5.62 e. -1.63

$\frac{13\cancel{\pi}}{6} \cdot \frac{180°}{\cancel{\pi}} = \boxed{390°}$ $\frac{3\pi}{4} \cdot \frac{180°}{\pi} = \boxed{135°}$ $-\frac{2\pi}{3} \cdot \frac{180°}{\pi} = \boxed{-120°}$ $5.62 \cdot \frac{180°}{\pi} = \boxed{322.00°}$ $-1.63 \cdot \frac{180°}{\pi} = \boxed{-93.39°}$

To convert degrees to radians, multiply by $\frac{\pi}{180°}$. → this is equivalent to 1

11. Convert each degree angle to radians. Answer in exact form.... No decimals.

a. 135° b. 540° c. −270°

$= 135° \cdot \frac{\pi}{180°} = \frac{135\pi}{180} = \boxed{\frac{3\pi}{4}}$

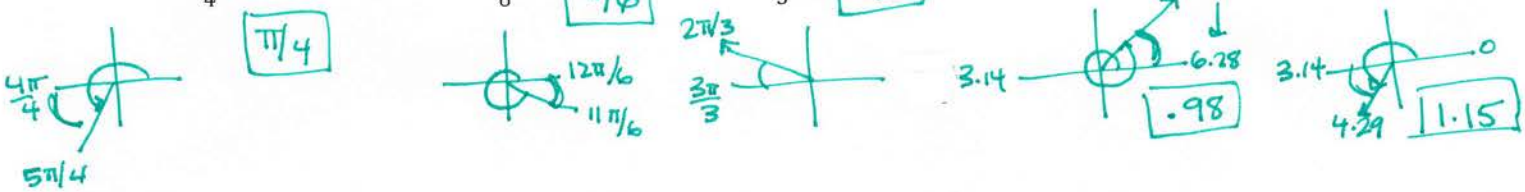
$540° \cdot \frac{\pi}{180°} = \frac{540\pi}{180} = \boxed{3\pi}$

$-270° \cdot \frac{\pi}{180°} = -\frac{270\pi}{180} = \boxed{-\frac{3\pi}{2}}$

Finding Trig ratios:

12. Find the measure of the reference angle for the given angles.

a. $\frac{5\pi}{4}$ $\boxed{\pi/4}$ b. $\frac{11\pi}{6}$ $\boxed{\pi/6}$ c. $\frac{2\pi}{3}$ $\boxed{\pi/3}$ d. 7.26 $\boxed{.98}$ e. 4.29 $\boxed{1.15}$



Find the exact value of the following. Remember quadrant and reference angles?

13. $\sin\frac{4\pi}{3} = \boxed{-\sqrt{3}/2}$

Q: III
ref∠: π/3 (60°)

14. $\tan\frac{7\pi}{6} = \boxed{\sqrt{3}/3}$

Q: III
ref∠: π/6 (30°)

15. $\cos\frac{3\pi}{4} = \boxed{-\sqrt{2}/2}$

Q: II
ref∠: π/4 (45°)

16. $\sec\frac{7\pi}{6} = \boxed{-\frac{2\sqrt{3}}{3}}$

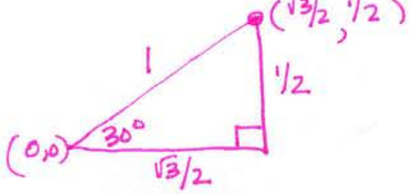
Q: III
ref∠: π/6 (30°)
$\cos 7\pi/6 = -\sqrt{3}/2$

17. $\cot\left(-\frac{\pi}{3}\right) = \boxed{-\sqrt{3}/3}$

Q: IV
ref∠: π/3 (60°)
$\tan(-\pi/3) = -\sqrt{3}$

Day 4: Axis Angles / Unit Circle

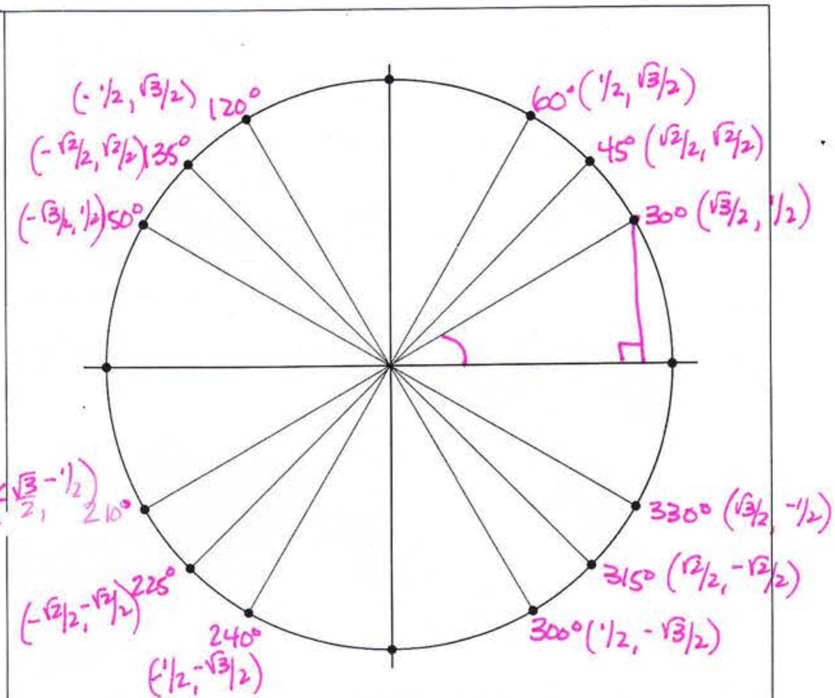
- Find the point of intersection for the unit circle and a 30° angle.



- What does the point of intersection between an angle and the unit circle tell you?

$\boxed{\begin{aligned} x &= \cos\theta \\ y &= \sin\theta \end{aligned}}$

if (x,y) is a point on Unit Circle

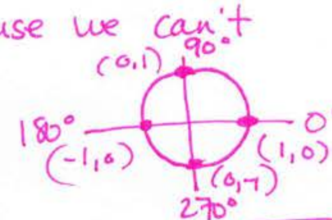


Axis angles:

Axis angles are angles in standard position whose terminal side lies on a coordinate axis.
The 4 primary axis angles measure 0°, 90°, 180°, and 270°.

****How can we find trig ratios for axis angles?**

* Use the point on the unit circle because we can't draw a reference triangle.



$\boxed{* \ \tan\theta = \frac{y}{x}}$

Find sin, cos, and tan for each given angle **without** using a calculator.

1. $\theta = 0$ $\boxed{\begin{aligned} \sin 0 &= 0 \\ \cos 0 &= 1 \end{aligned}}$ $\tan 0 = \frac{0}{1} = \boxed{0}$

2. $\theta = \frac{\pi}{2}$ $\boxed{\begin{aligned} \sin \pi/2 &= 1 \\ \cos \pi/2 &= 0 \\ \tan \pi/2 &= \tfrac{1}{0} = \text{undef.} \end{aligned}}$

3. $\theta = \pi$ $\boxed{\begin{aligned} \sin \pi &= 0 \\ \cos \pi &= -1 \\ \tan \pi &= 0 \end{aligned}}$

4. $\theta = \frac{3\pi}{2}$ $\boxed{\begin{aligned} \sin \tfrac{3\pi}{2} &= -1 \\ \cos \tfrac{3\pi}{2} &= 0 \\ \tan \tfrac{3\pi}{2} &= \text{undef} \end{aligned}}$

5. Find the following ratios **without a calculator**.
   a. sin 180° $\boxed{0}$
   b. tan (-90°) $\frac{-1}{0} = \boxed{\text{undef}}$
   c. sec 90° $= \boxed{\text{undef}}$ $\cos 90° = 0$
   d. csc 270° $= \boxed{\text{undef}}$ $\sin 270° = 0$

Remember:
The reciprocal of 0 is undefined **The reciprocal of undefined is** zero

Day 5: Inverse Trig functions

Solve the following equation. Make sure to give all possible answers.

$\sin\theta = \frac{1}{2}$ $\boxed{\theta = 30°, 150°, 390°, 510° \ldots}$

*all angles that have a sin of 1/2

Inverse Trig Functions:

- Used to find an angle measure when you know the ratio.
- Remember that functions only have one answer for each x-value.

	Definition	Notation	Restrictions (So it is a FUNCTION)
Inverse Sin	Finds the angle if you know the sin	$\sin^{-1}$ or arcsin	$-\pi/2 \le \theta \le \pi/2$ $-90° \le \theta \le 90°$
Inverse Cos	Finds the angle if you know the cos	$\cos^{-1}$ or arc cos	$0 \le \theta \le \pi$ $0° \le \theta \le 180°$
Inverse Tan	Finds the angle if you know the tan	$\tan^{-1}$ or arc tan	$-\pi/2 \le \theta \le \pi/2$ $-90° \le \theta \le 90°$

****Unless a problem indicates otherwise, use the given restrictions when using inverse trig functions.**

Examples:

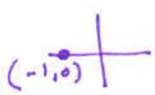
Evaluate the following **without** a calculator.

1. $\arcsin\left(-\frac{1}{2}\right)$

angle whose sin is $-1/2$

$\boxed{-30° \text{ or } -\pi/2}$

2. $\cos^{-1}(-1)$ $\boxed{180° \text{ or } \pi}$



3. $\arctan\sqrt{3}$ $\boxed{60° \text{ or } \pi/3}$

angle whose tan is $\sqrt{3}$

4. $\sin^{-1}\frac{\sqrt{3}}{2}$

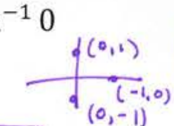
$\boxed{60° \text{ or } \pi/3}$

5. $\arccos(-2)$

angle whose cos is -2

$\boxed{\text{undefined}}$ *cos is adj/hyp can't be bigger than 1

6. $\tan^{-1} 0$ $\boxed{0° \text{ or } 0}$



7. $\arcsin\left(\sin\frac{5\pi}{3}\right)$ Q: IV ref∠: 60°

$= \arcsin(-\sqrt{3}/2)$

$= \boxed{-\pi/3}$

8. $\cos^{-1}(\cos\pi)$

$= \cos^{-1}(-1)$

$= \boxed{\pi}$

9. $\arcsin\left(\cos\frac{3\pi}{4}\right)$ Q II ref 45°

$= \arcsin(-\sqrt{3}/2)$

$= \boxed{-\pi/4}$

10. $\tan(\arctan(-5))$

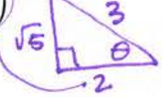
angle whose tan is -5

$=\tan(\text{angle whose tan is } -5)$

$= \boxed{-5}$:)

11. $\tan\left(\arccos\frac{2}{3}\right)$

$= \tan(\theta)$



$2^2 + x^2 = 3^2$

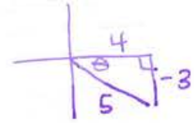
$x = \sqrt{5}$

$= \boxed{\frac{\sqrt{5}}{2}}$

12. $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$

$= \cos(\theta)$

$= \boxed{\frac{4}{5}}$



$(-3)^2 + x^2 = 5^2$

$x = 4$

Calculators: You can use the calculator to find angles using inverse functions. Remember, the calculator uses the restricted intervals when finding an angle. If you are looking for a different angle you will need to adjust the calculator answer.

Solve for the following angles using the given intervals.

13. $\cos\theta = 0.4 \qquad 0° \le \theta \le 180°$

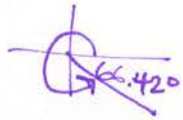
$\theta = \cos^{-1}(.4)$ $\boxed{\theta = 66.42°}$

Calc answer : 66.42°

looks between 0° & 180° – so no adjusting needed

14. $\cos\theta = 0.4 \qquad 270° \le \theta \le 360°$ Q IV

$\theta = \cos^{-1}(.4)$



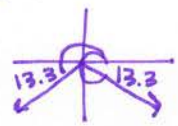
$\theta = 360° - 66.42°$
$= \boxed{293.58°}$

Calc answer: 66.42° ref< = 66.42°

looks between 0° and 180° – we need to adjust

15. $\sin\theta = -0.23 \qquad 180° \le \theta \le 360°$ Q III & IV

$\theta = \sin^{-1}(-.23)$



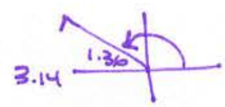
2 answers

$180° + 13.3° = \boxed{193.3°}$
$360° - 13.3° = \boxed{346.7°}$

Calc answer: –13.3° → ref< = 13.3°

looks between –90° and 90° so need to adjust.

16. $\tan\theta = -4.7 \qquad \frac{\pi}{2} \le \theta \le \pi$ Q II Radians



$\theta = 3.14 - 1.36 = \boxed{1.78}$

$\theta = \tan^{-1}(-4.7)$

Calc answer: –1.36 – ref< = 1.36

looks between –π/2 and π/2 So need to adjust