Unit 8 Notes / Secondary 3 Honors

Teacher Copy

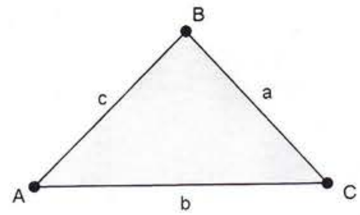
Day 1: Law of Sines

To **solve a triangle** means to find all the missing sides and angles.

An **oblique triangle** is a triangle that is NOT a right triangle.

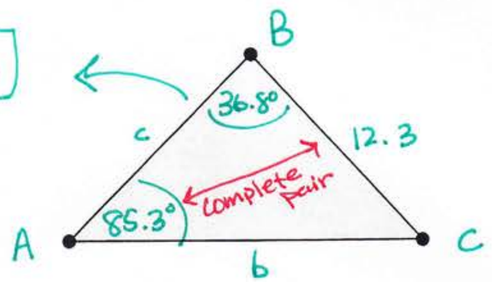
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \text{Or} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



1. Solve triangle ABC given A = 85.3° B = 36.8° a = 12.3 in.

AAS



$\frac{b}{\sin 36.8^\circ} = \frac{12.3}{\sin 85.3^\circ}$ $b = \frac{12.3 \sin 36.8^\circ}{\sin 85.3^\circ} = 7.39 = b$

$C = 180^\circ - 85.3^\circ - 36.8^\circ = 57.9^\circ$

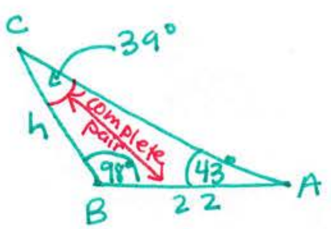
$\frac{c}{\sin 57.9^\circ} = \frac{12.3}{\sin 85.3^\circ}$

$c = \frac{12.3 \sin 57.9^\circ}{\sin 85.3^\circ} = 10.45$

$$\boxed{\begin{array}{l} b = 7.39 \\ C = 57.9^\circ \\ c = 10.45 \end{array}}$$

2. A pole tilts *toward* the sun at an 8° angle from the vertical, and it casts a 22 foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 43°. How tall is the pole?

ASA



$C = 180^\circ - 98^\circ - 43^\circ = 39^\circ$

$\frac{22}{\sin 39^\circ} = \frac{h}{\sin 43^\circ}$

$h = \frac{22 \sin 43^\circ}{\sin 39^\circ} \approx \boxed{23.84 \text{ feet}}$

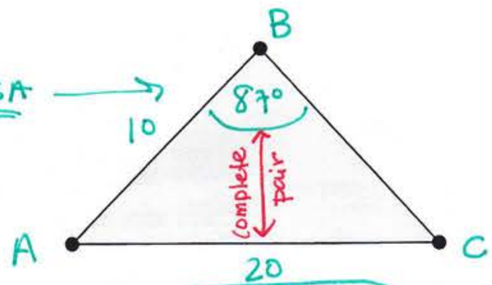
* You need to know a "complete pair" in order to have enough info for Law of Sines.

3. SSA → not a congruence theorem - possible to have 1 Δ, 2 Δ's, or 0 Δ's

Three Cases (The Ambiguous Case)

a. Solve triangle ABC given B = 87°, c = 10 in., b = 20 in.

* SSA →



$\boxed{C = 29.95^\circ \quad A = 63.05^\circ \quad a = 17.85}$

$$\frac{\sin C}{10} = \frac{\sin 87^\circ}{20}$$

$$\sin C = \frac{10 \sin 87^\circ}{20}$$

$$C = \sin^{-1}\left(\frac{10\sin 87^\circ}{20}\right) = \underset{\text{calc}}{29.95^\circ},\ \cancel{\underset{180^\circ-\text{calc}}{150.05^\circ}}$$

* Check to see if each angle would "fit" in Δ for angle C.

150.05° too big for Δ

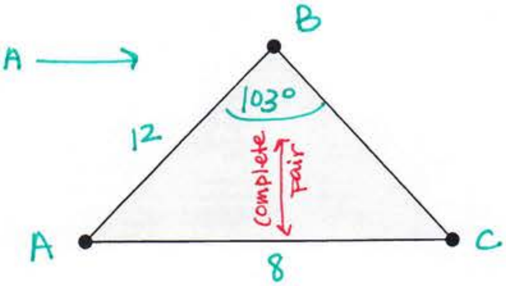
So only 1 Δ

$$\frac{a}{\sin 63.05^\circ} = \frac{20}{\sin 87^\circ}$$

$$a = 17.85$$

b. Solve triangle ABC given B = 103°, c = 12 in., b = 8 in.

* SSA →



$$\frac{\sin C}{12} = \frac{\sin 103^\circ}{8}$$

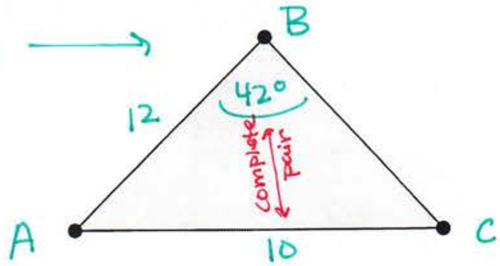
Can't have a sin value greater than 1 or less than -1

$$C = \sin^{-1}\left(\frac{12\sin 103^\circ}{8}\right) = \sin^{-1}(1.462) = \underline{\text{no solution}}$$

$\boxed{\text{No } \Delta \text{ possible}}$

c. Solve triangle ABC given B = 42°, c = 12 in., b = 10 in.

* SSA →



$$\frac{\sin C}{12} = \frac{\sin 42^\circ}{10}$$

$$C = \sin^{-1}\left(\frac{12\sin 42^\circ}{10}\right) = \underset{\text{calc}}{53.41^\circ},\ \underset{180^\circ-\text{calc}}{126.59^\circ}$$

Both angles "fit" in Δ for C so 2 Δ's

Δ1	Δ2
C = 53.41°	C = 126.59°
A = 84.29°	A = 11.41°
a = 14.87	a = 2.96

$$\frac{a}{\sin 84.29^\circ} = \frac{10}{\sin 42^\circ}$$

$$\frac{a}{\sin 11.41^\circ} = \frac{10}{\sin 42^\circ}$$

<u>Day 2</u>: Law of Cosines

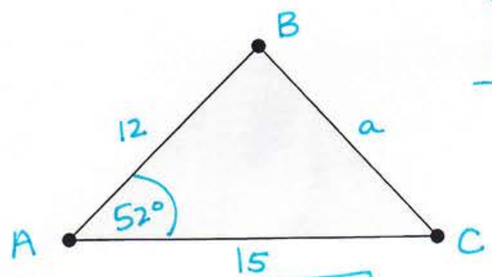
Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

1. Solve the triangle given c = 12 in. b = 15 in. A = 52°

SAS



$a^2 = 12^2 + 15^2 - 2(12)(15)\cos 52°$

$a = \sqrt{12^2 + 15^2 - 2(12)(15)\cos 52°} = 12.14$

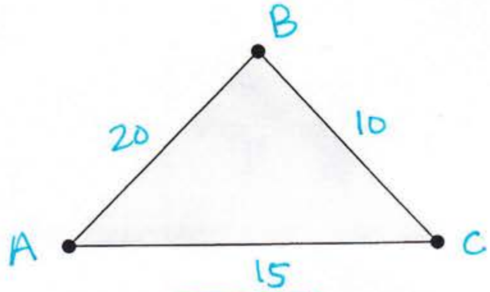
$\frac{\sin C}{12} = \frac{\sin 52°}{12.14} \qquad C = \sin^{-1}\left(\frac{12\sin 52°}{12.14}\right) = 51.16°$

$a = 12.14$
$C = 51.16°$
$B = 76.84°$

* You only need to use Law of Cosines to find first piece of Δ. You then have enough info to switch to Law of Sines. :)

2. Solve the triangle given a = 10 in. b = 15 in. c = 20 in.

SSS



$10^2 = 20^2 + 15^2 - 2(20)(15)\cos A$

$10^2 - 20^2 - 15^2 = -2(20)(15)\cos A$

$\frac{10^2 - 20^2 - 15^2}{-2(20)(15)} = \cos A$

$\frac{-525}{-600} = \cos A \qquad A = \cos^{-1}\left(\frac{-525}{-600}\right) = 28.96°$

$\frac{\sin B}{15} = \frac{\sin 28.96°}{10}$

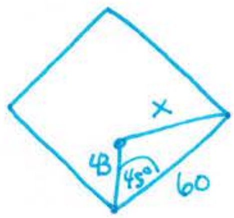
$B = \sin^{-1}\left(\frac{15\sin 28.96°}{10}\right) = 46.58°$

$A = 28.96°$
$B = 46.58°$
$C = 104.46°$

<u>Remember:</u> DO NOT use Law of Sines to find the largest angle in your triangle!!

If angle is obtuse $\sin^{-1}$ can't find it for you $(-90° \le \theta \le 90°)$

3. The pitcher's mound on a women's softball field is 43 feet from home plate and the distance between the bases is 60 feet. (The pitcher's mound is *not* halfway between home plate and second base.) How far is the pitcher's mound from first base?



$x^2 = 43^2 + 60^2 - 2(43)(60)\cos 45°$

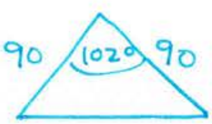
$x = 42.43$ feet

Finding the AREA of a triangle:

If you have a SAS situation given in a triangle, you can find the area using the following formula.

$$Area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

4. Find the area for a triangle having two sides of lengths 90 inches and an included angle of 102°.



$Area = \frac{1}{2}90(90)\sin 102°$

$= \boxed{3961.50 \text{ in}^2}$

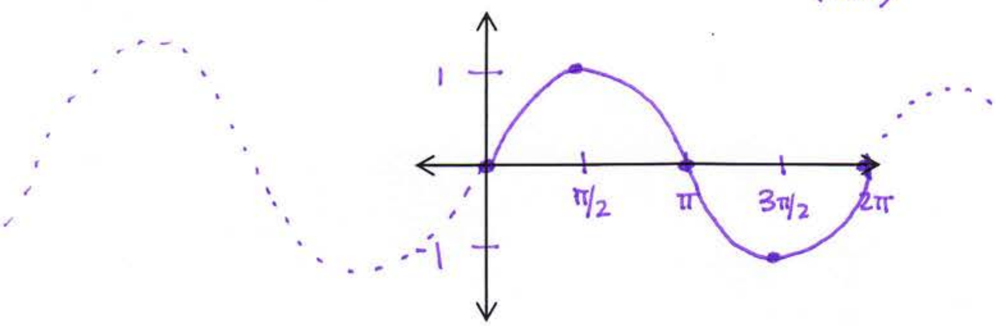
Day 3: Graphs of Sin and Cos Functions

Amplitude: a positive value that represents half the distance between the max and min values of the function.

Period: the length of one full cycle of the graph....how far the graph goes before it repeats itself.

The Sine Function: $y = sinx$

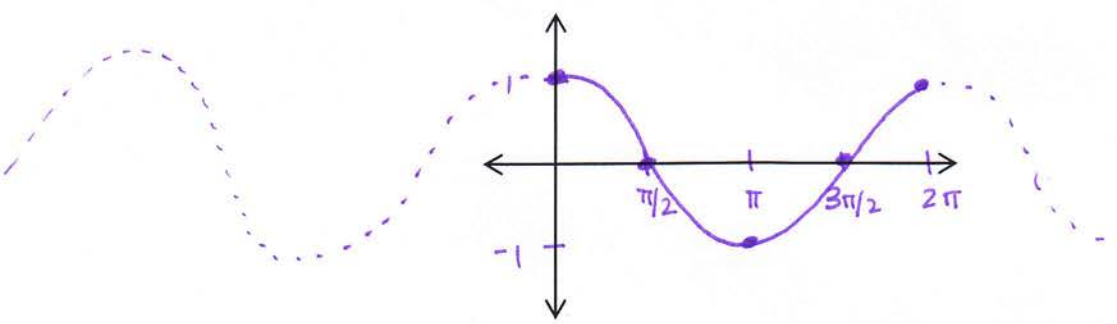
Amplitude	Period	Domain	Range	Symmetry?	y-intercept	x-intercepts
1	2π	$\mathbb{R}$	$-1 \le y \le 1$	origin (ODD)	(0,0)	$0, \pi, 2\pi....$



x	y (sinx)
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

The Cosine Function: $y = cos\, x$

Amplitude	Period	Domain	Range	Symmetry?	y-intercept	x-intercepts
1	2π	$\mathbb{R}$	$-1 \le y \le 1$	y-axis (EVEN)	(0,1)	$\pi/2, 3\pi/2...$



x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

Graphing Transformations:

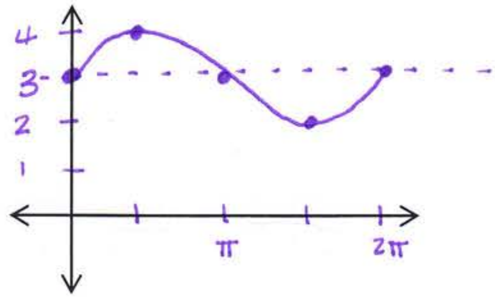
$y = a \sin b(x - c) + d$ $\qquad$ $y = a \cos b(x - c) + d$

a = Vertical stretch / Shrink (amplitude) Vertical reflection if negative	b = Horizontal stretch / Shrink (period) H. reflection if negative
d = Vertical shift (same direction as sign)	c = Horiz. Shift (phase shift) (opp. direction of sign)

* **Remember to graph one complete cycle with starting, middle, and ending x & y-values labeled.**

Vertical Shift

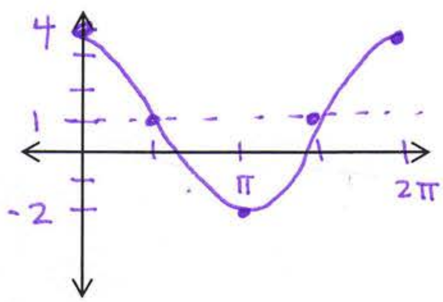
1. Graph $y = \sin x + 3$ ↑ 3



Vertical Stretch or Shrink

2. Graph $y = 3\cos x + 1$ ↑ 1

Amplitude = 3 (go up 3 and down 3 from center line)

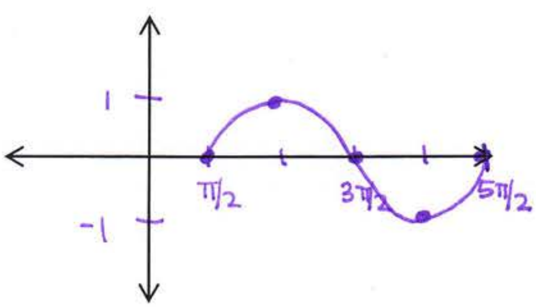


Horizontal Shift (Known as a <u>phase shift</u> for trig functions)

3. $y = \sin(x - \frac{\pi}{2})$ right $\pi/2$

Start $\pi/2$ → End $5\pi/2$

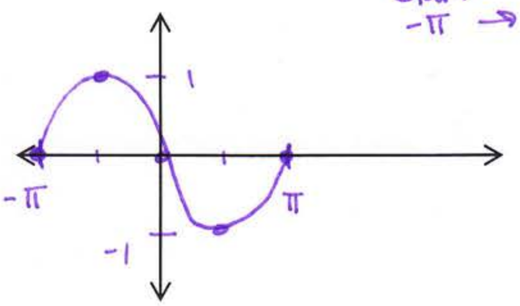
$(0 + \pi/2)$ $(2\pi + \pi/2)$



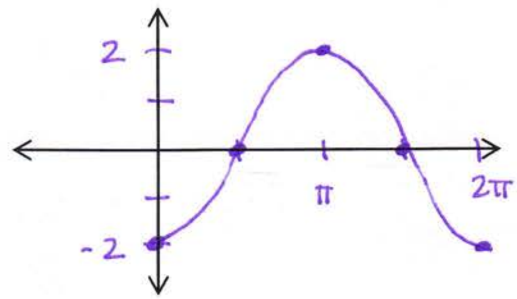
Examples: Graph one complete cycle of the following.

4. $y = \sin(x + \pi)$ ← π

Start End
−π → π

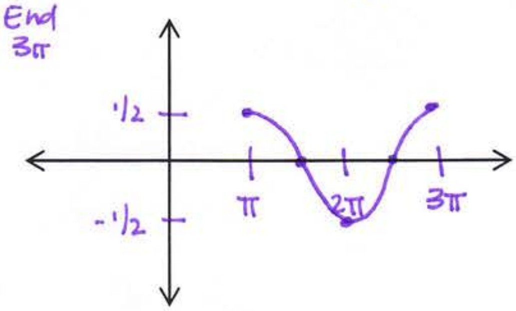


5. $y = -2\cos x$ reflect amp = 2

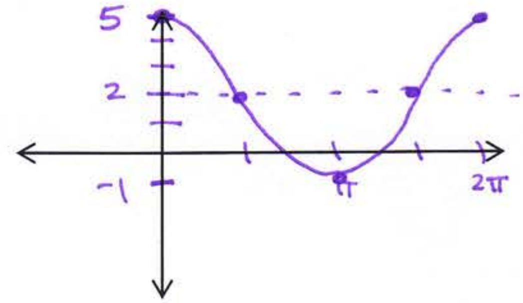


6. $y = \frac{1}{2}\cos(x - \pi)$ → π amp = 1/2

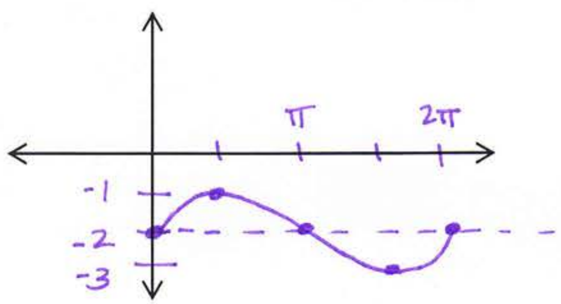
Start End
π → 3π



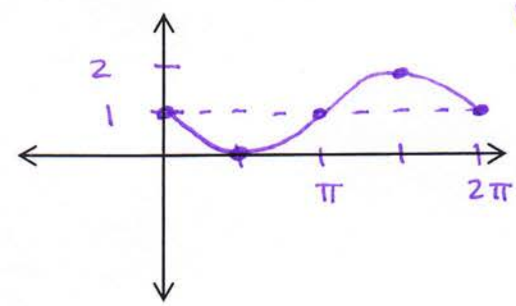
7. $y = 3\cos x + 2$ ↑2 amp = 3



8. $y = -2 + \sin x$ ↓2



9. $y = 1 - \sin x$ ↑1 reflect



Day 4: **Graphing Sin, Cos, and Tan functions**

Remember graph transformations? What does a, b, c, and d do to your graph?

$$y = a\sin b(x-c) + d \qquad\qquad y = a\cos b(x-c) + d$$

Horizontal Stretch or Shrink (Plus a horizontal reflection if b is negative.)

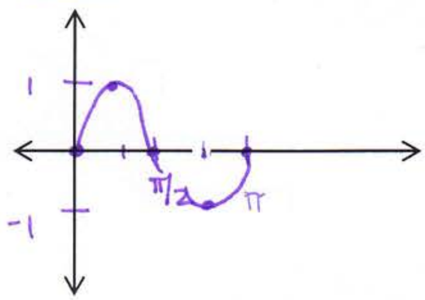
- "b" ***change*** the period of the trig functions
- New period = $\dfrac{original\ \ period}{|b|}$

Graph one cycle of the following.

1. $y = \sin(2x)$ period $= \frac{2\pi}{2} = \pi$

Start → End
$0 \rightarrow \pi$

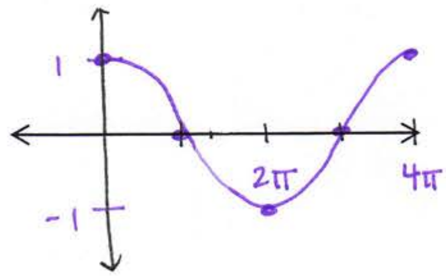
Period: π



2. $y = \cos\left(\frac{x}{2}\right)$ $\frac{2\pi}{1/2} = 4\pi$

Start $0 \rightarrow$ End 4π

period: 4π

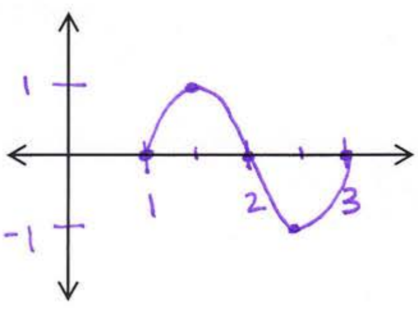


If your graph has a period change and a phase shift, do the period change first.

3. $y = \sin\pi(x - 1)$ period $= \frac{2\pi}{\pi} = 2$

S → E
$0 \rightarrow 2$

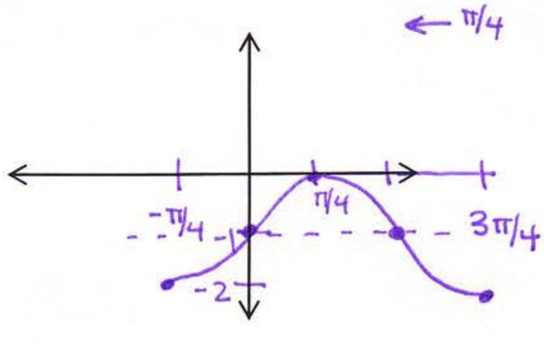
$\rightarrow 1$ S → E
$1 \rightarrow 3$



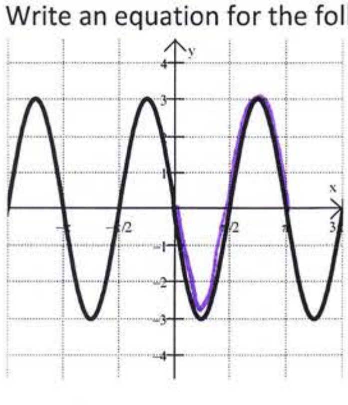
4. $y = -\cos\left(2x + \frac{\pi}{2}\right) - 1$ ↓1

$y = -\cos 2(x + \pi/4) - 1$ period $= \frac{2\pi}{2} = \pi$

$\leftarrow \pi/4$ S → E
$\cancel{0} \rightarrow \pi$
$-\pi/4 \rightarrow \frac{3\pi}{4}$



5. Write an equation for the following graph.



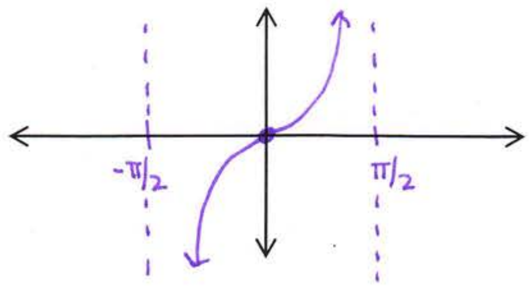
Sine
$y = -3\sin 2x$

Cosine
$y = 3\cos 2(x - \pi/4)$

The Tangent Function: $y = tanx$

Period	Domain	Range	Symmetry?	Asymptotes	y-intercept	x-intercepts
π	$\mathbb{R}$, $x \neq \pi/2 \pm \pi n$	$\mathbb{R}$	Origin	$\pi/2$, $-\pi/2$ etc.	$(0,0)$	$0, \pi, \ldots$

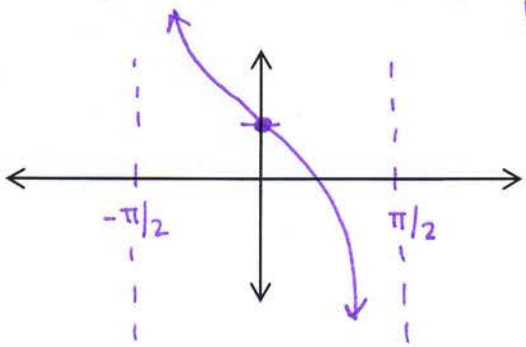
NO Amp



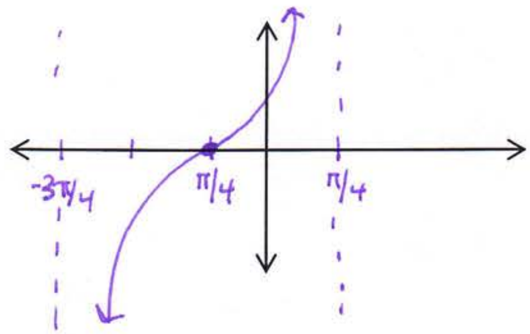
x	y
0	0
$\pi/4$	1
$\pi/2$	undef
$-\pi/4$	-1
$-\pi/2$	undef

Examples: Graph one complete cycle of each function.

6. $y = -3\tan x + 1$ ↑1 reflect



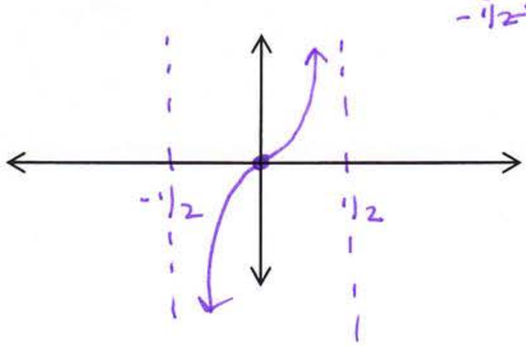
7. $y = \tan(x + \frac{\pi}{4})$ ← $\pi/4$



8. $y = \tan \pi x$

period $= \frac{\pi}{\pi} = 1$

S E
$-1/2 \to 1/2$

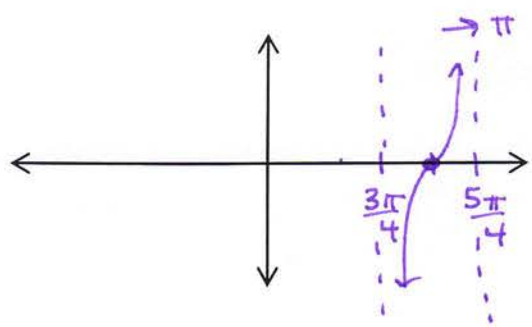


9. $y = \tan 2(x - \pi)$

period $= \pi/2$

S E
$-\pi/4 \to \pi/4$

S
$\frac{3\pi}{4} \to \frac{5\pi}{4}$

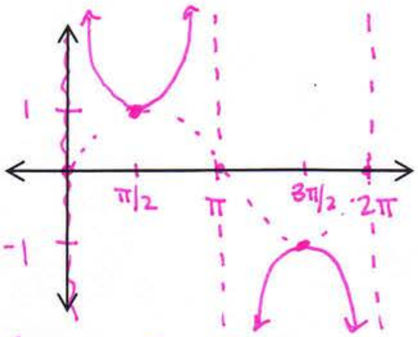


Day 5: Graphs of the reciprocal trig functions

The easiest way to graph reciprocal trig functions is to use the basic graphs and "take the reciprocal".

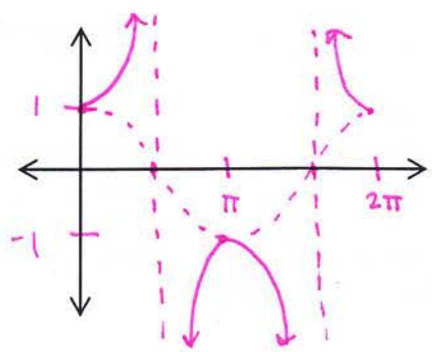
The Cosecant Function $y = \csc x$ (reciprocal of sin)

Period	Domain	Range	Symmetry?	Asymptotes
2π	$\mathbb{R}, x \neq \pi \ldots$	$y \geq 1$ $y \leq -1$	Origin	$0, \pi, 2\pi \ldots$



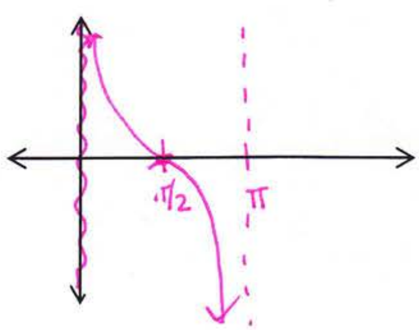
The Secant function $y = \sec x$ (recip of cos)

Period	Domain	Range	Symmetry?	Asymptotes
2π	$\mathbb{R}, x \neq \pi/2 \ldots$	$y \geq 1, y \leq -1$	y-axis	$\pi/2, 3\pi/2 \ldots$



The Cotangent Function $y = \cot x$ (recip of tan)

Period	Domain	Range	Symmetry?	Asymptotes



Graph one complete cycle of the following:

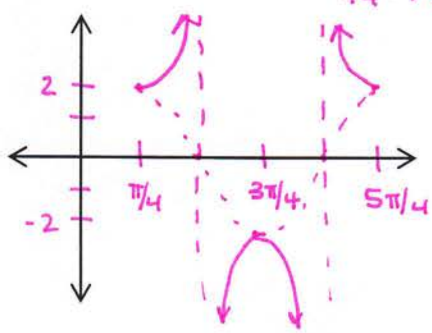
reflect

period $= \frac{\pi}{\pi} = 1$

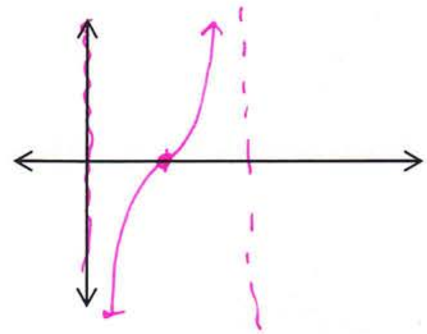
S → E
0 → 1

1. $y = 2\sec\left(x - \frac{\pi}{4}\right)$ → $\pi/4$

S → E
$\pi/4 \rightarrow 5\pi/4$



2. $y = -2cot(\pi x)$

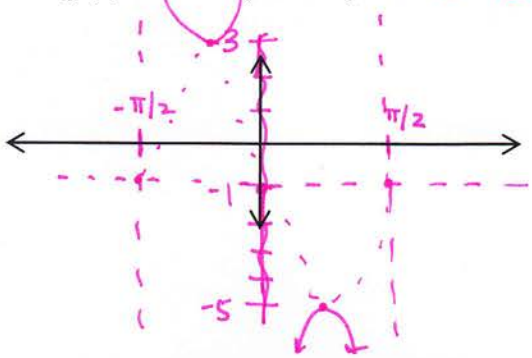


3. $g(x) = 4\, csc(2x + \pi) - 1$ $= 4\csc 2(x + \pi/2) - 1$

period $= \frac{2\pi}{2} = \pi$ S → E: $0 \rightarrow \pi$

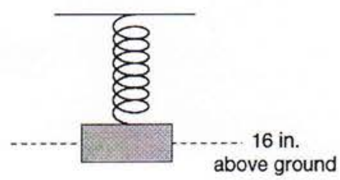
$\leftarrow \pi/2$ $(-\pi/2 \rightarrow \pi/2)$

$\downarrow 1$

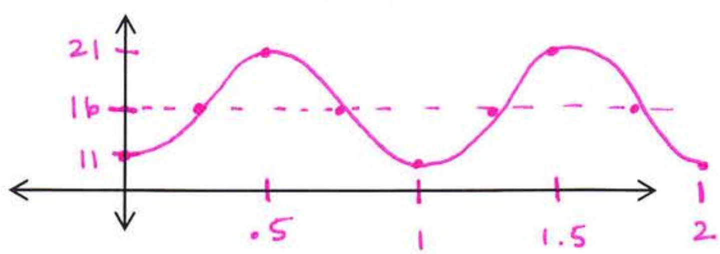


7. An object suspended from a spring is pulled 5 inches below its resting position and released, causing the object to bounce up and down once every second. At rest, the object's height above the ground is 16 inches.

Suppose that the object bounces up to 5 inches above its resting height and then back down to 5 inches below its resting height without stopping on every bounce.



a. Sketch and label the graph of the function modeling the bouncing object over time. Show at least 2 bounces.



b. Write the equation of the function.

$y = -5\cos 2\pi x + 16$

g. Graph your function on your graphing calculator (in RAD mode) for $0 \leq t \leq 5$. Determine the height of the object at time 3 seconds.

11 inches

h. Determine the time on the interval $0 \leq t \leq 1$ when the height of the object is 18 inches.

$\boxed{.32 \text{ \& } .68 \text{ sec}}$