Unit 12
Solving Systems of Equations

Day 1

Solve Linear Systems by Graphing

System of equations: a set (or collection) of equations
2 or more eq'ns w/ 2 or more variables

System of linear equations: the graphs are lines ($y=mx+b$) ($Ax+By=C$)
x is to the first power

The solution of a system is: any x & y that makes the eq'n true (the intersection)

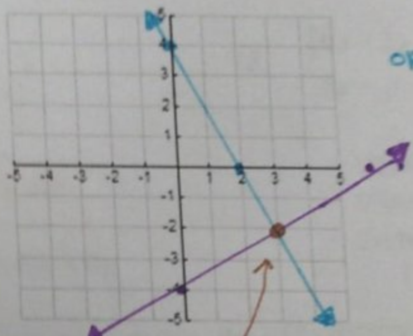
To solve a linear system by graphing:

(x,y) one sol'n — no sol'n — ∞ infinitely many sol'ns

- ✓ Graph each line: use a straightedge!
- ✓ Identify the point that is on both lines (intersection)

Example.

1. Solve the system by graphing:
$\begin{cases} 2x+y=4 \\ 2x-3y=12 \end{cases}$ (2,0)(0,4) (6,0)(0,-4)



OR solve for y: $y=-2x+4$

OR solve for y: $y=\frac{2}{3}x-4$

$2x+y=4$

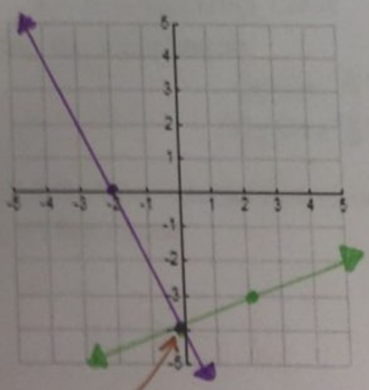
The solution is: $\boxed{(3,-2)}$

the point of intersection $x=3$ $y=-2$ (3,-2)

To check your solution:
$2(3)+(-2)=4$ ✓
$2(3)-3(-2)=12$ ✓

Examples. Solve by graphing. Check your solution.

2. $\begin{cases} y=\frac{1}{2}x-4 \\ 4x+2y=-8 \end{cases}$
(-2,0)(0,-4)



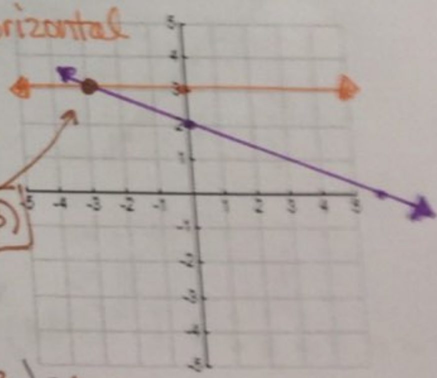
$y=\frac{1}{2}x-4$

$4x+2y=-8$

$\boxed{(0,-4)}$

$-4=\frac{1}{2}(0)-4$ ✓
$4(0)+2(-4)=-8$ ✓

3. $\begin{cases} y=3 \\ x+3y=6 \end{cases}$ y = horizontal
(6,0)(0,2)

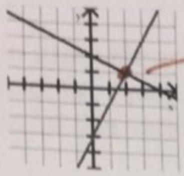
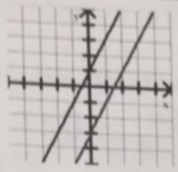
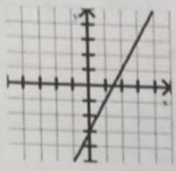


$\boxed{(-3,3)}$

$3=3$ ✓
$(-3)+3(3)=6$ ✓

For systems of two-variable linear equations, the following cases are possible:

classify system with these words
↓

	Lines intersect: one solution (x,y) if find x, then MUST find y... so find it!	Consistent / independent · different slopes
	Lines are parallel: no solution no sol'n when solving.. # = different #	Inconsistent · same slope · different y-intercept
	Lines Coincide: infinitely many solutions ∞ when solving... # = same #	Consistent / dependent · same slope · same y-intercept

Examples. Write each equation in $y = mx + b$ form. Then classify the system. DO NOT SOLVE.

4. $\begin{cases} y = 2x + 4 \\ 2x - y = 6 \end{cases}$ → $y = 2x - 6$

- same slope (2)
- different y-int (no sol'n)

inconsistent

$0 = -2$ ↙ If solving, it would look like this

5. $\begin{cases} 6x - 2y = 8 \\ 3x - y = 4 \end{cases}$ $2y = 6x - 8$ → $y = 3x - 4$; $y = 3x - 4$

- same slope (3)
- same y-int (-4)
- same line (∞)

consistent/dependent

$0 = 0$ ↙ If solving, it would look like this

Application:

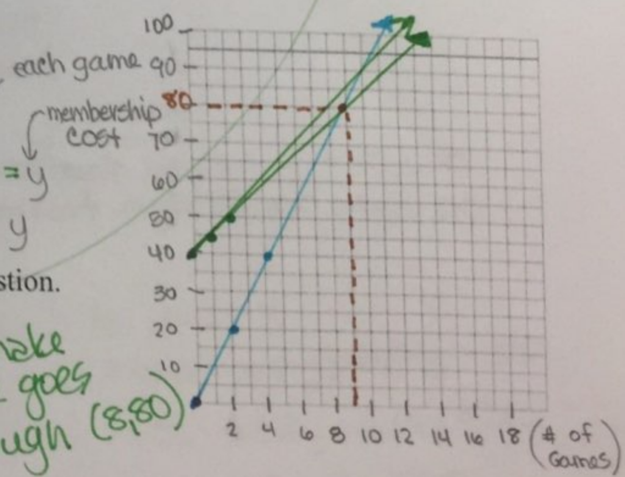
6. A soccer league offers two options for membership plans. Option A includes an initial fee of $40 and costs $5 for each game played. Option B costs $10 for each game played. After how many games will the total cost of the two options be the same?

Identify the variables: x = Each Game
y = membership cost

Write two equations from the information.

Option A: $40 + 5x = y$
Option B: $10x = y$

Graph both equations and find the intersection to answer the question. Be sure to show scales on both axes.



Redo make sure goes through (8,80)

Assignment 12.1

Next year... questions examples that have ∞, & no sol'n

Day 2

Solve Linear Systems using Substitution

To solve a system using the substitution method:

- ✓ Solve one of the equations for a variable (a variable with a coefficient of 1, if possible)
- ✓ Substitute for that variable in the *other* equation.
- ✓ Solve.
- ✓ Use the value to find the value of the other variable.
- ✓ Check the solution in the original system.

Easier to solve the eq'n that has just x or just y

Examples. Solve the system using substitution.

1. $\begin{cases} x = y \\ 5x - 3y = 10 \end{cases}$ $x=y$ ... substitute y for every x value

$5(y) - 3y = 10$ CLT

$2y = 10$ Solve for y

$\{y = 5\}$ Must find x! plug y into eq'n you didn't use to solve for y.

$\{x = 5\}$

$\boxed{(5,5)}$

$5=5$ ✓

$5(5)-3(5)=10$ ✓

2. $\begin{cases} 4x + 3y = -2 \\ x + 5y = -9 \end{cases}$ $\rightarrow x = -5y - 9$

$\boxed{(1,-2)}$

$4-6=-2$ ✓

$1-10=-9$ ✓

$4(-5x-9) + 3y = -2$ Distribute

$-20x - 36 + 3y = -2$ CLT

$-17y = 34$ Solve for y

$\{y = -2\}$

$\{x = -5(-2) - 9 = 1\}$

3. $\begin{cases} \frac{1}{5}x + \frac{1}{2}y = 8 \\ x + y = 20 \end{cases}$ $x = -y + 20$

$\frac{1}{5}(-y + 20) + \frac{1}{2}y = 8$

$5\left(-\frac{1}{5}y + 4 + \frac{1}{2}y = 8\right)$ Multiply by 5

$2\left(-y + 20 + \frac{5}{2}y = 40\right)$ Multiply by 2

$-2y + 40 + 5y = 80$ CLT

$3y = 40$ $\{y = \frac{40}{3}\}$

$\boxed{\left(\frac{20}{3}, \frac{40}{3}\right)}$

$\frac{1}{5}\left(\frac{20}{3}\right) + \frac{1}{2}\left(\frac{40}{3}\right) = 8$ ✓ plug in calc.

$\frac{20}{30} + \frac{40}{3} = \frac{60}{3} = 20$ ✓

$3\left(x = -\frac{40}{3} + 20\right)$ Multiply by 3

$3x = -40 + 60$ $3x = 20 \rightarrow x = \frac{20}{3}$

4. $\begin{cases} 2x + 4y = 6 \\ x = 3 - 2y \end{cases}$

$2(3 - 2y) + 4y = 6$

$6 - 2y + 4y = 6$

$2y = 0$

$\{y = 0\}$

$x = 3 - 2(0)$

$\{x = 3\}$

$\boxed{(3,0)}$

$6 + 0 = 6$ ✓

$3 = 3 - 0$ ✓

Application:

5. Louise invested a <u>total of $10,000</u> into two accounts. One account pays 2.5% simple interest and the other pays 4% simple interest. After one year, the <u>interest totaled $310</u>. How much did she invest in each account?

Define the two variables.

x = amount in 2.5% account

y = amount in 4% account

Write a system of equations that describes the situation.

investment + investment = total investment $\rightarrow x + y = 10{,}000$

interest + interest = total interest $\rightarrow 0.025x + 0.04y = 310$

Solve the system using substitution to answer the question.

$y = 10{,}000 - x$

$0.025x + 0.04(10{,}000 - x) = 310$

$0.025x + 400 - 0.04x = 310$

$-0.015x = -90$

$x = 6000$ $y = 4000$

$x = 10{,}000 - y$

$0.025(10{,}000 - y) + 0.04y = 310$

$250 - 0.025y + 0.04y = 310$

$0.015y = 60$

$y = 4000$ $x = 6000$

Assignment 12.2

$6000 in 2.5% account

$4000 in 4% account

Day 3

Solve Linear Systems using Elimination

Remember:
- If you find x, then find y → (x,y)
- check your answers

To solve a system using the elimination method:

- ✓ Pick a variable to eliminate.
- ✓ Multiply one or both equations by a number so that when the equations are added together, the chosen variable disappears.
- ✓ Add the equations together.
- ✓ Solve for the remaining variable.
- ✓ Substitute that value into one of the original equations to find the value of the other variable.
- ✓ Check the solution in the original system.

(x,y)   # ≠ # no sol'n   # = # ∞

Examples. Solve the system using the method of elimination.

1. $\begin{cases} 2x+3y=8 \\ 2x-4y=-6 \end{cases}$ Eliminate x

$-(2x-4y=-6)$

add
$2x+3y=8$
$-2x+4y=6$
$7y=14$
$y=2$

$2x+3(2)=8$
$2x+6=8$
$-6\;\;-6$
$2x=2$
$x=1$

$\boxed{(1,2)}$ $2+6=8$ ✓ $2-8=-6$ ✓

2. $\begin{cases} 8x+2y=4 \\ -2x+3y=13 \end{cases}$ Eliminate X

$4(-2x+3y=13)$

add
$8x+2y=4$
$-8x+12y=52$
$14y=56$
$y=4$

$8x+2(4)=4$
$8x+8=4$
$8x=-4$
$\frac{8x}{8}=\frac{-4}{8}$
$x=-\frac{1}{2}$

$\boxed{(-\frac{1}{2},4)}$ $-4+8=4$ ✓ $1+12=13$ ✓

3. $\begin{cases} 3x-6y=9 \\ -4x+7y=-16 \end{cases}$ Eliminate x

$4(3x-6y=9)$
$3(-4x+7y=-16)$

add
$12x-24y=36$
$-12x+21y=-48$
$-3y=-12$
$y=4$

$3x-6(4)=9$
$3x-24=9$
$3x=33$
$x=11$

$\boxed{(11,4)}$ $33-24=9$ ✓ $-44+28=-16$ ✓

4. $\begin{cases} \frac{1}{3}x-\frac{1}{2}y=\frac{1}{6} \\ 3x-2y=9 \end{cases}$ LCD = 6   Eliminate x

$6\left(\frac{2x}{6}\right)-6\left(\frac{3y}{6}\right)=6\left(\frac{1}{6}\right)$

$3(2x-3y=1)$
$-2(3x-2y=9)$

add
$6x-9y=3$
$-6x+4y=-18$
$-5y=-15$
$y=3$

$3x-2(3)=9$
$3x-6=9$
$3x=15$
$x=5$

$\boxed{(5,3)}$ $\frac{5}{3}-\frac{3}{2}=\frac{10-9}{6}=\frac{1}{6}$ ✓ $15-6=9$ ✓

Identifying inconsistent systems (no solution) and consistent/dependent systems (infinitely many solutions):

// # ≠ # no sol'n   = # = # ∞

Examples. Solve the system:

5. $\begin{cases} 3x-2y=8 \\ -6x+4y=5 \end{cases}$ Eliminate x

$2(3x-2y=8)$

add
$6x-4y=16$
$-6x+4y=5$
$0=21$
$\# \neq \#$
no sol'n

$\boxed{\text{inconsistent}}$

6. $\begin{cases} x-2y=9 \\ 3x-6y=27 \end{cases}$ Eliminate x

$-3(x-2y=9)$

add
$-3x+6y=-27$
$3x-6y=27$
$0=0$
$\# = \#$
∞

$\boxed{\text{consistent/dependent}}$

If you have a choice, which algebraic method is better (substitution or elimination)?

substitution if single x or y ⋮ Elimination for all others

Application:

7. At a pizza restaurant it costs \$4 to make a small pizza that sells for \$12, and it costs \$6 to make a large pizza that sells for \$15. In one week, the restaurant spent a total of \$1100 making pizzas and sold all of them for \$2910. How many small pizzas were sold? (Define variables, write a system, and solve using elimination.)

Define Variables:

x = small pizza

y = large pizza

make + make = make $\xrightarrow{-3}$ $(4x+6y=1100)$ eliminate x

sold + sold = sold → $12x+15y=2910$

$-12x-18y=-3300$

add $12x+15y=2910$

$-3y=-390$

$y=130$

$4x+6(130)=1100$

$4x+780=1100$

$4x=320$

$x=80$

→ how many x?

$\boxed{\text{80 small pizzas were sold}}$

*we could have eliminated y to get x...

Assignment 12.3

Day 4

Solve Linear Systems with Three Variables

- ✓ Label the equations A, B, and C.
- ✓ Pick a variable to eliminate.
- ✓ Write down the procedure needed to eliminate the variable from two of the equations in terms of A, B, and C, and then carry out the procedure. Repeat with two different equations.
- ✓ Solve the resulting 2-variable system using either substitution or elimination.
- ✓ Back-substitute to find the value of the third variable.
- ✓ Answers are written as an ordered triple. (x, y, z)

Examples. Solve the systems of equations. Substitution b/c z = 3

1. A $x+2y-z=8$ B $y+3z=7$ C $z=3$

$x+2(-2)-(3)=8 \rightarrow x-4-3=8 \rightarrow x=8+7 \rightarrow x=15$

$y+3(3)=7 \rightarrow y=-2$

$\boxed{(15,-2,3)}$

$15-4-3=8$ ✓

$-2+9=7$ ✓

$3=3$ ✓

2. A $x+y+z=6$ B $2x-y+z=3$ C $3x+y-z=2$

Eliminate z

B+C

$2x-y+z=3$

$3x+y-z=2$

$5x=5$

$x=1$

Eliminate y

A+B

$x+y+z=6$

$2x-y+z=3$

$3x+2z=9$

Plug x=1 Solve for z

$3(1)+2z=9$

$2z=6$

$z=3$

$(1)+y+(3)=6$

$y+4=6$

$y=2$

$\boxed{(1,2,3)}$

$1+2+3=6$ ✓

$2-2+3=3$ ✓

$3+2-3=2$ ✓

3. $\begin{cases} A & x-2y+3z=9 \\ B & -x+3y+z=-2 \\ C & 2x-5y+5z=17 \end{cases}$

Eliminate x

A+B

$x-2y+3z=9$
$-x+3y+z=-2$
$y+4z=7$

2B+C

$-2x+6y+2z=-4$
$2x-5y+5z=17$
$y+7z=13$

Eliminate y

$y+4z=7$
$-(y+7z=13)$
$-3z=-6$
$(z=2)$

$y+7(2)=13$
$(y=-1)$

$x-2(-1)+3(2)=9$
$x+2+6=9$
$(x=1)$

$\boxed{(1,-1,2)}$

ch: $1+2+6=9$ ✓
$-1-3+2=-2$ ✓
$2+5+10=17$ ✓

Three-variable systems can be inconsistent or dependent also.

Solve the systems:

4. $\begin{cases} A & x-3y+z=1 \\ B & 2x-y-2z=2 \\ C & x+2y-3z=-1 \end{cases}$

Eliminate x

$-2A+B$

$-2x+6y-2z=-2$
$2x-y-2z=2$
$5y-4z=0$

$B+(-2)C$

$2x-y-2z=2$
$-2x-4y+6z=2$
$-5y+4z=4$

$5y-4z=0$
$-5y+4z=4$
$0=4$

no sol'n

$\boxed{\text{inconsistent}}$

5. $\begin{cases} A & x+y-3z=-1 \\ B & y-z=0 \rightarrow z=y \\ C & -x+2y=1 \end{cases}$

$x+y-3(y)=-1$

$x-2y=-1$
$-x+2y=1$
$0=0$
∞

$\boxed{\text{consistent/ dependent}}$

Application:

6. A small corporation borrowed \$775,000 to expand its software line. Some of the money was borrowed at 8%, some at 9%, and some at 10%. How much was borrowed at each rate if the annual interest was \$66,000 and the amount borrowed at 8% was four times the amount borrowed at 10%?

x = money borrowed at 8%
y = money borrowed @ 9%
z = money borrowed @ 10%

A $x+y+z=775{,}000$

$100(.08x+.09y+0.1z=66{,}000)$

Multiply by 100 to get rid of decimal

B $8x+9y+10z=6{,}600{,}000$

C $x=4z$ want to eliminate y! b/c of this...

Multiply 1st eq'n by -9

$-9x-9y-9z=-6{,}975{,}000$
$8x+9y+10z=6{,}600{,}000$
$-x+z=-375{,}000$

substitute $x=4z$

$-(4z)+z=-375{,}000$
$-3z=-375{,}000$

z = 125,000	@ 10%
x = 500,000	@ 8%
y = 150,000	@ 9%

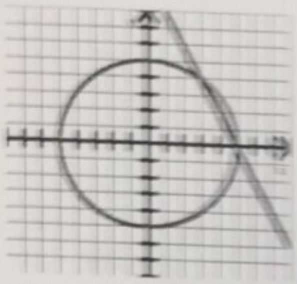
Day 5

Solve Non-linear Systems

When two lines intersect, it is at a single point (if it's an independent system). But what about a line and a parabola? What about two circles? When non-linear systems are solved, there is often more than one solution (point of intersection).

Remember: eq'n of circle $(x-h)^2+(y-k)^2=r^2$ where center is (h,k) & radius is r

If we graph the following system, we can see that we will have 2 solutions. Use substitution to solve the system algebraically to find the exact solutions of the system.



$$\begin{cases} x^2+y^2=25 \\ 2x+y=10 \end{cases} \quad y=-2x+10$$

$x^2+(-2x+10)^2=25$

$x^2+(4x^2-40x+100)=25$

$5x^2-40x+75=0$

$5(x^2-8x+15)=0$

$5(x-5)(x-3)=0$

$x=5$, $y=0$ $\quad$ $x=3$, $y=4$

$\boxed{(5,0)}$ & $\boxed{(3,4)}$

Non-linear systems can often be solved with elimination, but sometimes substitution is necessary.

Examples.

1. Solve by substitution

$$\begin{cases} x^2+y^2=53 \\ -x+y=5 \end{cases} \quad y=x+5$$

$x^2+(x+5)^2=53$

$x^2+(x^2+10x+25)=53$

$2x^2+10x-28=0$

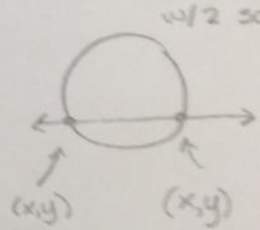
$2(x^2+5x-14)=0$

$2(x+7)(x-2)=0$

$x=-7$, $y=-2$ $\quad$ $x=2$, $y=7$

$\boxed{(-7,-2) \text{ \& } (2,7)}$

circle & a line w/2 sol'ns



2 circles/2 pts

2. Solve by elimination

$$\begin{cases} 3x^2+2y^2=36 \\ 2(4x^2-y^2=4) \end{cases}$$

$8x^2-2y^2=8$

$11x^2=44$

$x^2=4$

$x=\pm 2$

$x=2$, $y=\sqrt{12}$ $\quad$ $x=-2$, $y=\sqrt{12}$

$x=2$, $y=-\sqrt{12}$ $\quad$ $x=-2$, $y=-\sqrt{12}$

$\boxed{(2,\overset{2\sqrt3}{\sqrt{12}}) \text{ \& } (2,\overset{-2\sqrt3}{-\sqrt{12}}) \text{ \& } (-2,\overset{2\sqrt3}{\sqrt{12}}) \text{ \& } (-2,\overset{-2\sqrt3}{-\sqrt{12}})}$

$3(4)+2y^2=36$

$2y^2=24$

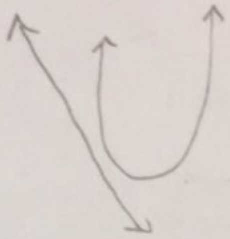
$y^2=12$

$y=\pm\sqrt{12}$

$=\pm 2\sqrt3$

could place $2\sqrt3$ in place of $\sqrt{12}$

3. Solve by either method: $\begin{cases} -x+y=4 \\ x^2+y=3 \end{cases}$ $y=x+4$



line & parabola w/ no real sol'ns.

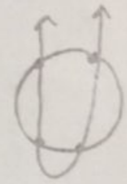
$x^2+(x+4)=3$

$x^2+x+1=0$

$a=1 \quad b=1 \quad c=1$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}=\dfrac{-1\pm\sqrt{-3}}{2}$ imaginary $\boxed{\text{no real sol'n}}$

circle & parabola w/ 4 sol'ns



4. Solve $\begin{cases} x^2+y^2=13 \\ x^2-y=7 \end{cases}$

$-(x^2-y=7)$

$-x^2+y=-7$

$y^2+y=6$

$y^2+y-6=0$

$(y+3)(y-2)=0$

$x=$
$y=-3$

$x=$
$y=2$

$x^2-(-3)=7$
$x^2=4$
$x=\pm2$

$x^2-(2)=7$
$x^2=9$
$x=\pm3$

$x=\pm2$
$y=-3$

$x=\pm3$
$y=2$

$\boxed{\begin{array}{c}(2,-3)\ \&\ (-2,-3) \\ (3,2)\ \&\ (-3,2)\end{array}}$

5. The sum of two numbers is 7 and the sum of their squares is 29. Find the numbers.

$x+y=7 \rightarrow y=7-x$

$x^2+y^2=29$

$x^2+(7-x)^2=29$

$x^2+(49-14x+x^2)=29$

$2x^2-14x+20=0$

$2(x^2-7x+10)=0$

$2(x-5)(x-2)=0$

$x=5$
$y=2$

$x=2$
$y=5$

$\boxed{5\ \&\ 2}$

Assignment 12.5

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Day 6
Unit 12 Review
Assignment 12.6

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Day 7
Unit 12 Test ***All late/absent assignments due for Unit 12***

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