Unit 4 Begin Unit w/ 228-229 "Play is our Work" → orally (just ask students to respond to questions)

Polynomial Expressions and Equations

Day 1 Pg 231 will help w/ average rate of change

Analyzing Polynomial Functions

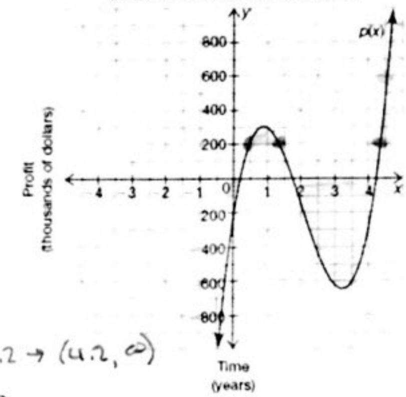
interval notation

Play is Our Work

The polynomial function *p(x)* models the profits of Zorzansa, a video game company, from its original business plan through its first few years in business.

The cubic function *p(x)* models Zorzansa's total profits over the first five years of business.

Zorzansa's Profits Over Years 0 - 5



1. Use the graph to estimate when Zorzansa's achieved each of the following.
   a. $600,000 profit 4.5 years (estimate the x-value @ $600,000)
   b. $200,000 profit 0.5 years & 1.3 years & 4.4 years
   c. profit greater than $200,000 $0.5 < x < 1.3 \rightarrow (0.5, 1.3)$; $x > 4.4 \longrightarrow (4.4, \infty)$
   d. the company is losing money ↳ $0 < x < 0.2 \rightarrow (0, 0.2)$; $1.75 < x < 4.2 \rightarrow (1.75, 4.2)$
   e. the company is making a profit ↳ $0.2 < x < 1.75 \rightarrow (0.2, 1.75)$ & $x > 4.2 \rightarrow (4.2, \infty)$

Pg 231

Average Rate of Change: the ratio of the change in the dependent variable to the change in the independent variable over a specified time interval. The formula for average rate of change is: $\dfrac{f(b)-f(a)}{b-a}$ for the interval (a, b).

$b - a$ represents the change in the input values of the function *f*. $x_2 - x_1$

$f(b) - f(a)$ represents the change in the output values of the function *f* as the input values change from *a* to *b*. $y_2 - y_1$

2. a. Find the average rate of change of Zorzansa's profit for the time interval $x = 1.5$ to $x = 2.5$.

   @ $x = 1.5$, $y = 200$ → $a = 1.5$ $f(a) = 200$
   @ $x = 2.5$, $y = -400$ → $b = 2.5$ $f(b) = -400$
   $\dfrac{-400 - 200}{2.5 - 1.5} = \dfrac{-600}{1} = \boxed{-600}$

   b. Multiply the average rate of change in part *a* by $1000. Why is this necessary to reflect the true average rate of change?

   $(-600)(1000) = \boxed{-600{,}000}$ b/c the y-values are measured in "thousands of dollars"

   c. Why is the average rate of change negative over this interval?

   the profit is decreasing

   d. What does the average rate of change mean in this problem situation?

   It represents the average change in the profit over the time interval of 1.5 years & 2.5 years.

3. Determine the average rate of change of Zorzansa's profits for the time interval $x = 1$ to $x = 4.5$. Explain the meaning of this average rate of change.

   $f(1) \approx 300$
   $f(4.5) \approx 600$
   $\dfrac{f(4.5) - f(1)}{4.5 - 1} = \dfrac{300 - 600}{3.5} = \dfrac{-300}{3.5} = -85.714 * 1000 = -85714$

   Avg Rate of Change $\approx \boxed{-86000}$

Assignment 4.1

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During the time interval of 1 year to 4.5 year, the profits (on average) were decreasing by about 86000. Even though from years 3-4 increase

Day 2

Polynomial Division

pg 237: Long Division Example
pg 239: #3a Example
#3b Assignment
Then we don't worry about long division again :)

The Fundamental Theorem of Algebra states that every polynomial equation of degree n must have n zeros. This means that every polynomial can be written as the product of n factors of the form $(ax+b)$. For example,

$$2x^2 - 3x - 9 = (2x+3)(x-3).$$

A factor of an integer divides into that integer with a remainder of zero. This process can also help determine other factors. For example, knowing that 5 is a factor 115, you can determine that 23 is also a factor since $\frac{115}{5} = 23$. In the same manner, factors of polynomials can divide into that polynomial without a remainder.

pg 237

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. The process is similar to long division with integers.

1. Compare:

Divisor, quotient, Divident

$$9\overline{)472} = 52 \quad -45,\ 22,\ -18,\ 4$$

Remainder

$= 52R4$ or $52\frac{4}{9}$

vs.

$$x+3\overline{)2x^2+x-15} = 2x-5$$

$-(2x^2+6x)$
$-5x-15$
$-(-5x-15)$
0

Remainder = 0

The Process:
① $\frac{2x^2}{x} = 2x$ } quotient
② $2x(x+3)$
③ Subtract
④ $\frac{-5x}{x} = -5$ } quotient
⑤ $-5(x+3)$
⑥ Subtract

2. Is 9 a factor of 472?

NO b/c there is a remainder ($R \neq 0$)

Is $x+3$ a factor of $2x^2+x-15$? Explain your reasoning.

yes b/c no remainder ($R=0$)

3. Determine the quotient $q(x)$ for each of the following. Which divisors are factors of the dividends?

place holder for x^2 b/c $x^3\ x^2\ x^1\ x^0$

a. $\dfrac{3x^3-12x^2+x-4}{x-4}$

$q(x) =$

$$x-4\overline{)3x^3-12x^2+x-4} = 3x^2+0+1$$

$-(3x^3-12x^2)$
$0+x-4$
$-(x-4)$
0

$\boxed{q(x) = 3x^2+1}$ $\boxed{\text{Factor}}$ b/c $R=0$

ch: $(x-4)(3x^2+1) = 3x^3-12x^2+x-4$ ✓

b. $(9x^3+8x-2) \div (3x+2)$

$$3x+2\overline{)9x^3+0x^2+8x-2} = 3x^2-2x+4$$

$-(9x^3+6x^2)$
$-6x^2+8x$
$-(-6x^2-4x)$
$12x-2$
$-(12x+8)$
-10

$\boxed{q(x) = 3x^2-2x+4}$

$\boxed{\text{Not a Factor}}$ b/c $R \neq 0$

pg 241 Example of Synthetic Division

Long division is sometimes necessary but the process can be inefficient and time consuming.

Synthetic division is a shortcut method for dividing a polynomial by a linear factor of the form $(x-r)$. This method requires fewer calculations and less writing because it leaves out the variables and uses only multiplication and addition. But since it leaves out the variables, once the process has been completed, the quotient and remainder have to be interpreted and written out.

Use synthetic division to divide the problem in example 3c: $\frac{3x^3-12x^2+x-4}{x-4}$. Identify the quotient.

Is the quotient the same as when long division is used?

$x-4=0$
$x=4$

$$\begin{array}{r|rrrr} 4 & 3 & -12 & 1 & -4 \\ & \downarrow & 12 & 0 & 4 \\ \hline & 3 & 0 & 1 & \boxed{0} \end{array}$$

$(3\times4)=12$
$0\times4=0$
$1\times4=4$

quotient ↖ remainder

Quotient: $3x^2+0x+1$

$\boxed{q(x)=3x^2+1}$

factors: $(x-4)(3x^2+1)$

remainder = 0

4. Use synthetic division to find $\frac{g(x)}{f(x)}$. Write the answer as $f(x)\cdot q(x)+R$ where R is the remainder.

a. $g(x)=2x^4-4x^3+50x+6$
$f(x)=x+3$

Place holder $x^4\, x^3\, x^1\, x^0$

$$\begin{array}{r|rrrrr} -3 & 2 & -4 & 0 & 50 & 6 \\ & \downarrow & -6 & 30 & -90 & 120 \\ \hline & 2 & -10 & 30 & -40 & \boxed{126} \end{array}$$

remainder

$f(x)\cdot q(x)+R$

$\boxed{(x+3)(2x^3-10x^2+30x-40)+126}$

b. $g(x)=3x^3+x^2+x-2$
$f(x)=3x-2$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & 1 & 1 & -2 \\ & \downarrow & 2 & 2 & 2 \\ \hline & 3 & 3 & 3 & \boxed{0} \end{array}$$

remainder

$f(x)\cdot q(x)+R$

$\boxed{(3x-2)(3x^2+3x+3)}$

The Remainder Theorem

$$\frac{dividend}{divisor}=quotient+\frac{remainder}{divisor} \qquad \text{OR} \qquad dividend=(divisor)(quotient)+remainder$$

5. Given $p(x)=x^3+8x-2$ and $\frac{p(x)}{(x-3)}=x^2+3x+17+\frac{49}{x-3}$,

a. Identify $q(x)$ and verify that it is the quotient.

$x-3=0$
$x=3$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 8 & -2 \\ & \downarrow & 3 & 9 & 51 \\ \hline & 1 & 3 & 17 & \boxed{49} \end{array}$$

quotient ↖ remainder

$\boxed{q(x)=x^2+3x+17}$

- b. Identify the remainder R.

$R=\boxed{49}$

- c. Evaluate $p(3)$.

$p(3)=(3)^3+8(3)-2=\boxed{49}$

d. What is the relationship between the remainder when the function is divided by $x-3$ and the function evaluated at 3?

$\boxed{\text{Same}}$

Remainder Theorem: When any polynomial equation or function $f(x)$, is divided by a linear factor $(x-a)$, the remainder is $R = f(a)$, or the value of the function when $x = a$.

6. Given $p(x) = x^3 + 6x^2 + 5x - 12$ and $\frac{p(x)}{x-2} = x^2 + 8x + 21 + \frac{30}{x-2}$, Ren says that $p(-2) = 30$ and Pam says that $p(2) = 30$. Without performing any calculations, who is correct? Explain your reasoning.

Pam → $P(2) = 30$; if $x-2$ is the divisor, you are finding $p(2)$

$x-2=0$
$x=2$

7. The function $f(x) = 4x^2 + 2x + 9$ generates the same remainder when divided by $(x-a)$ and $(x-2a)$, where a is not equal to 0. Calculate the value(s) of a.

$x=a$ $x=2a$

$$\begin{array}{c|ccc} a & 4 & 2 & 9 \\ & \downarrow & 4a & 2a+4a^2 \\ \hline & 4 & 2+4a & \boxed{4a^2+2a+9} \end{array}$$

$$\begin{array}{c|ccc} 2a & 4 & 2 & 9 \\ & \downarrow & 8a & 16a^2+4a \\ \hline & 4 & 8a+2 & \boxed{16a^2+4a+9} \end{array}$$

$4a^2 + 2a + 9 = 16a^2 + 4a + 9$

solve for a:

$0 = 12a^2 + 2a$ GCF: $2a$

$0 = 2a(6a+1)$

$2a = 0$ $6a+1=0$

$\cancel{a=0}$ $\boxed{a = -1/6}$

b/c $a \neq 0$

Assignment 4.2

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Day 3

The Factor Theorem

Factor Theorem: A polynomial has a linear polynomial as a factor if and only if the remainder is zero.

iff

In other words, $f(x)$ has $(x-a)$ as a factor if and only if $f(a) = 0$.

Examples:

1. Find two different ways to show that $(x-7)$ is a factor of the polynomial $f(x) = x^3 - 10x^2 + 11x + 70$.

① $$\begin{array}{c|cccc} 7 & 1 & -10 & 11 & 70 \\ & \downarrow & 7 & -21 & -70 \\ \hline & \underbrace{1 \quad -3 \quad -10}_{\text{quotient}} & & & \boxed{0} \end{array}$$

Show $R=0$

— OR —

② Show $f(7) = 0$

$f(7) = (7)^3 - 10(7)^2 + 11(7) + 70$

$= 343 - 490 + 77 + 70 = 0$ ✓

2. You can use the $\boxed{\text{quotient}}$ to continue to factor the polynomial $f(x) = x^3 - 10x^2 + 11x + 70$. Factor the quotient and write $f(x)$ as a product of three linear factors.

$q(x) = x^2 - 3x - 10$

* Find factors of this quotient

$0 = (x-5)(x+2)$

$f(x) = (x-7)(x-5)(x+2)$

3. Use synthetic division to help find the unknown coefficient, k, in the function below: $k=$

$f(x)=2x^4+x^3-14x^2+kx-6$ if $(x-3)$ is a factor

If $(x-3)$ is a factor, then the remainder must = zero!

3	2	1	-14	k	-6
	↓	6	21	21	63+3k
	2	7	7	21+k	$\boxed{3k+57}$

remainder

$3k+57=0$

$3k=-57$

$\boxed{k=-19}$

4. Is $(3x+4)$ a factor of $f(x)=3x^3+13x^2+18x+8$?

$x=-\frac{4}{3}$

$-\frac{4}{3}$	3	13	18	8
	↓	-4	-12	-8
	3	9	6	$\boxed{0}$

remainder

$\boxed{\text{yes}}$ b/c the remainder is zero (R=0)

5. Is $f(x)=(x-3)(x+5)(x+2)(x-1)$ the factored form of $f(x)=x^4+3x^3-15x^2-19x+30$?

x-3:	3	1	3	-15	-19	30
		↓	3	18	9	-30
		1	6	3	-10	$\boxed{0}$
x+5:	-5	↓	-5	-5	10	
		1	1	-2	$\boxed{0}$	
x+2:	-2	↓	-2	2		
		1	-1	$\boxed{0}$		

remember this is a quotient

↳ x-1

$\boxed{\text{yes}}$ b/c all the remainders are equal to zero!

Factoring Higher Order Polynomials

Remember, you can use the difference of squares when you have a binomial of the form a^2-b^2. The binomial $a^2-b^2=(a+b)(a-b)$. Sometimes you will have to factor more.

6. x^2-64

$\boxed{(x-8)(x+8)}$

7. x^4-16

$(x^2-4)(x^2+4)$

$\boxed{(x-2)(x+2)(x^2+4)}$

☆ Factor Completely!

Polynomials with 4 terms can often by factored by grouping two terms together. The technique is called *factoring by grouping.* Make sure to use parentheses when grouping two terms together.

8. $x^3+2x^2+7x+14$

GCF: x^2 GCF: 7

$x^2(x+2)+7(x+2)$

GCF: $(x+2)$

$\boxed{(x+2)(x^2+7)}$

9. $3k^3-18k^2+k-6$

GCF: $3k^2$ GCF: 1

$3k^2(k-6)+1(k-6)$

At this point you know you did it right b/c GCF is $(k-6)$

$\boxed{(k-6)(3k^2+1)}$

Fundamental Thm of Algebra: should have 4 factors

10. $x^4-4x^3-x^2+4x$

$x^3(x-4)-x(x-4)$

$(x-4)(x^3-x)$ Factor completely! GCF: x

$(x-4)x(x^2-1)$ Factor completely! Diff of squares

$\boxed{x(x-4)(x+1)(x-1)}$

1 2 3 4

11. $3t^3-12t^2+5t-20$

$3t^2(t-4)+5(t-4)$

$\boxed{(t-4)(3t^2+5)}$

sometimes, the factor is a double factor as is the case w/ the t^2 term.

Assignment 4.3

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Day 4

Finding All Zeros of Polynomials

some of the zeros can be imaginary /complex (which always occur in pairs)

Remember, the zeros of a function are the solutions of $f(x)=0$ (factors). And the fundamental theorem of algebra says that for an *n*th degree function there are exactly *n* zeros.

Sometimes you can find all the zeros of polynomials by **factoring** and using the zero property.

Examples. Factor completely and find all the zeros:

1. $f(x)=x^3-3x^2-4x+12$

$0=x^2(x-3)-4(x-3)$

$0=(x-3)(x^2-4)$

$0=(x-3)(x-2)(x+2)$

$\boxed{\text{zeros: } 3,2,-2}$

2. $g(x)=3x^3-12x^2+9x$

$0=3x(x^2-4x+3)$

$0=3x(x-3)(x-1)$

$\boxed{\text{zeros: } 0,3,1}$

If you know a factor or zero of a polynomial function, you can find the rest of the factors and zeros using **synthetic division and working with the quotient.**

Example.

3 zeros

3. Given $(x-8)$ is a factor of $f(x)=x^3-7x^2-10x+16$ write $f(x)$ in factored form and find all the zeros.

8	1	−7	−10	16
	↓	8	8	−16
	1	1	−2	$\boxed{0}$

quotient

$(x-8)(x^2+x-2)$

$\boxed{(x-8)(x+2)(x-1)=f(x)}$

$\boxed{\text{zeros: } 8,-2,1}$

What if you are not given any factors or zeros? Should you start randomly choosing numbers and testing them to see if they divide evenly into the polynomial? This is a situation when the ***Rational Zero Theorem*** becomes useful.

Rational #s:

1 $-\frac{3}{2}$ 0.75 $\cancel{\sqrt{3}}$ $\cancel{\pi}$

irrational

Rational Zero Theorem: All rational zeros (roots) of a polynomial function with integer coefficients are of the form $\frac{p}{q}$, where p is a factor of the constant term, and q is a factor of the leading coefficient.

Examples.

4. Determine all possible rational zeros for the function $f(x) = 3x^5 - 12x^3 + 11x - 15$

q p

$-15: \pm1, \pm3, \pm5, \pm15$

-15
1 −15
3 −5
5 −3
15 −1

5. Complete each step to factor and solve $x^4 + x^3 - 7x^2 - x + 6 = 0$.
   a. Determine all the possible rational solutions.

   $\boxed{6: \pm1, \pm2, \pm3, \pm6}$

   b. Use synthetic division to determine an actual solution.

1	1	1	−7	−1	6
	↓	1	2	−5	−6
	1	2	−5	−6	$\boxed{0}$

   $(x-1)$ is a factor b/c remainder is zero $(R=0)$

   c. Rewrite the polynomial as a product of its quotient and linear factor.

   $(x-1)(x^3 + 2x - 5x - 6$

   d. Repeat steps a-c for the cubic quotient.

2	1	2	−5	−6
	↓	2	8	6
	1	4	3	$\boxed{0}$

   $(x-2)$ is a factor b/c $R=0$

   e. Factor the new quotient and find the other two solutions.

   $x^2 + 4x + 3 \Rightarrow (x+3)(x+1)$

   e. List all the solutions. $\boxed{\text{zeros: } 1, 2, -3, -1}$

6. Using the same steps in the previous problem, determine all zeros for $f(x) = x^3 + 3x^2 - 6x - 8$.

$-8: \pm1, \pm2, \pm4, \pm8$

~~1~~	1	3	−6	−8
	↓	1	4	−2
	1	4	−2	$\boxed{-10}$

b/c $R \neq 0$

2	1	3	−6	−8
		2	10	8
	1	5	4	$\boxed{0}$

$(x-2)$

$f(x) = (x-2)(x^2 + 5x + 4)$

or $0 = (x-2)(x+4)(x+1)$

$\boxed{\text{zeros: } 2, -4, -1}$

Complex zeros

Some of the zeros may be complex $\nearrow a+bi$. *These zeros cannot be found by factoring*. But if you can find a quadratic factor that doesn't factor, it can be solved using the quadratic formula or by taking the square root of both sides.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples.

7. Given that 3 is a solution of the equation $x^4 - 4x^3 + 19x^2 - 64x + 48 = 0$, find all real and complex solutions.

$$\begin{array}{r|rrrrr} 3 & 1 & -4 & 19 & -64 & 48 \\ & & 3 & -3 & 48 & -48 \\ \hline & 1 & -1 & 16 & -16 & \boxed{0} \end{array}$$

$f(x) = (x-3)(x^3 - x^2 + 16x - 16)$

$0 = (x-3)[x^2(x-1) + 16(x-1)]$

$0 = (x-3)(x-1)(x^2+16)$

$(x-3)(x-1)(x^2+16) = 0$

$x-3=0$ $\quad x=3$

$x-1=0$ $\quad x=1$

$x^2+16=0$ $\quad x^2=-16$ $\quad x=\pm 4i$

$\boxed{x = 3, 1, 4i, -4i}$

zeros
sol'ns
roots

8. Find all real and complex zeros of the following functions:

a. $m(x) = x^4 - 5x^2 - 36$

Try factoring:
treat x^4 like x^2

Ask: 2 Terms
1. GCF
2. are both terms square rootable w/a minus sign in between? then $(+)(-)$

$m(x) = (x^2-9)(x^2+4)$

$0 = (x+3)(x-3)(x^2+4)$

$x=-3$ $\quad x=3$ $\quad x^2+4=0$ $\quad x^2=-4$ $\quad x=\pm 2i$

$\boxed{\text{Zeros: } -3, 3, 2i, -2i}$

Ch: how many zeros should you have?

graph: does the graph cross the x axis @ $x=-3$ and $x=3$?

Does the graph cross the x axis @ any other points?

Then the other 2 points are complex.

b. $f(x) = x^3 + 2x^2 + 2x$

$a=1 \quad b=2 \quad c=2$

$0 = x\underbrace{(x^2+2x+2)}_{\text{Quadratic Formula}}$

$x=0$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$$

$$x = -\frac{2}{2} \pm \frac{\sqrt{-4}}{2}$$

$$x = -1 \pm \frac{2i}{2}$$

$x = -1 \pm i$

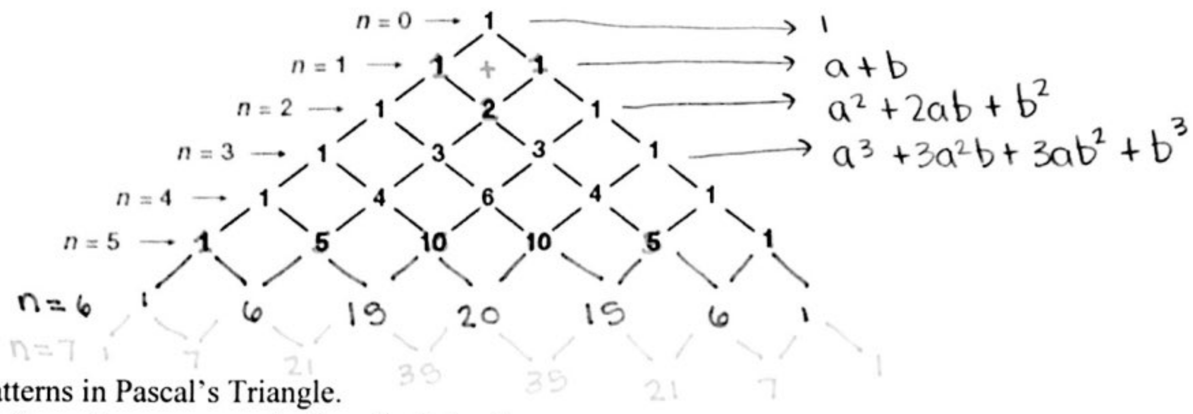
$\boxed{\text{Zeros: } 0, -1+i, -1-i}$

Day 5

Pascal's Triangle

There is an interesting pattern of numbers that makes up what is referred to as Pascal's Triangle.

The first six rows of Pascal's Triangle are shown, where n = 0 represents the first row, n = 1 represents the second row, and so on.



1. Analyze the patterns in Pascal's Triangle.
   a. Describe all the patterns you see in Pascal's Triangle.

      1 + 1 = 2 1 5 10 10 5 1 ↑ ↓

   b. Complete the rows for n = 6 and n = 7 in the diagram of Pascal's Triangle.

      when n is odd middle #s repeat (next to each other)
      when n is even ....

The patterns shown in Pascal's Triangle have many uses. For instance, you may have used Pascal's Triangle to calculate probabilities. Let's explore how you can use Pascal's Triangle to raise a binomial to a positive integer.

2. Multiply to expand each binomial. Write the final answer so that the powers of *a* are in descending order.
   a. $(a+b)^0$ $= \boxed{1}$

      b/c any # to the zero power = 1 !

   b. $(a+b)^1$

      $\boxed{a+b}$

   c. $(a+b)^2$

      $\boxed{a^2+2ab+b^2}$

   d. $(a+b)^3$

      $\boxed{a^3+3a^2b+3ab^2+b^3}$

3. Analyze the answers to example 2.
   a. Compare the coefficients of each product with the numbers shown in Pascal's Triangle. What do you notice?

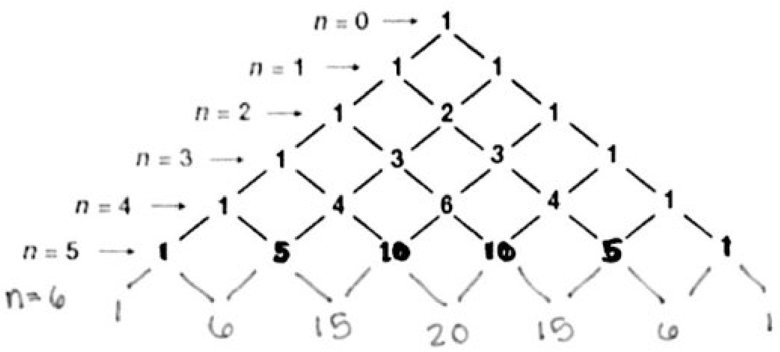
      $\boxed{\text{same}}$

   b. What do you notice about the exponents of the *a* and *b* variables in each expansion?

      as → first variable (a) go down by 1 power as → 2nd variable (b) go up by 1 power

   b. What do you notice about the sum of the exponents of the *a* and *b* variables in each expansion?

      $n=2 \rightarrow a^2 + 2a^1b^1 + b^2$ (2 2 2) | $n=3$ $a^3 + 3a^2b^1 + 3a^1b^2 + b^3$ (3 3 3 3) | each term add up to n



Binomial Expansion a↓ b↑

4. Use Pascal's Triangle to expand each binomial.

a. $(a+b)^5$

$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b. $(a+b)^6$

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Most binomials terms are not a and b. Consider $(2x+y)^3$ or $(4x-1)^5$. You can use the expansion for $(a+b)^n$ and then substitute for a and b to find the expansion in question. Be sure to use parentheses around each substitution.

5. Use Pascal's triangle and substitution to expand each binomial.

a. $(x+3)^4$ $a=x$ $b=3$

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(x)^4 + 4(x)^3(3) + 6(x)^2(3)^2 + 4(x)(3)^3 + b(3)^4$

x^4 $12x^3$ $6(9)x^2$ $4(27)x$ $(3)^4$

$\boxed{x^4 + 12x^3 + 54x^2 + 108x + 81}$

b. $(2-y)^5$

$a=2$ $b=-y$

$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(2)^5 + 5(2)^4(-y) + 10(2)^3(-y)^2 + 10(2)^2(-y)^3 + 5(2)(-y)^4 + (-y)^5$

$32 + -5(16)y + 10(8)y^2 + -10(4)y^3 + 10y^4 + -y^5$

$\boxed{32 - 80y + 80y^2 - 40y^3 + 10y^4 - y^5}$

Binomial Theorem

You could continue to find the rows of Pascal's Triangle to expand $(a+b)^{15}$, but it would be very time-consuming and tedious. There is another way to find the coefficients, involving combinations.

Combinations represent all the different ways that a set of r objects can be chosen from a set of n objects. Combinations are written as: $\binom{n}{r}$ or nCr . They are calculated with factorials: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

n Choose r

Fortunately, calculators can be used to find this number. You can evaluate combinations by going to the MATH menu, then choosing PRB. It is choice 3 in the PRB menu.

Use a calculator to find the following: $\binom{12}{3}$ and $\binom{10}{8}$. What happens if you try to evaluate $\binom{8}{10}$?

$\binom{12}{3}$ $\boxed{12}$ $\boxed{\text{MATH}}$ → → → PROB ↓↓ 3: nCr $\boxed{\text{enter}}$ 12C $\boxed{3}$ $\boxed{\text{enter}}$ $= \boxed{220}$

$\binom{10}{8} = \boxed{45}$

What is $\binom{10}{10}$?

$\binom{10}{10} = 1$ There's only one way to answer 10 questions on a 10 question test

$\binom{8}{10} = \boxed{0}$ can't have 10 choices if all there is to choose from is 8

The Binomial Theorem uses combinations to find the coefficients of a binomial expansion instead of Pascal's triangle. It's very helpful when expanding binomials raised to larger powers, such as $(a+b)^{10}$.

Binomial Theorem: $(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$

Use the Binomial Theorem to expand each binomial.

1. $(a+b)^4$ $\binom{4}{0}a^4 + \binom{4}{1}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{3}ab^3 + \binom{4}{4}b^4$

   $n=4$ 1   4   6   4   1 ← Do these look familiar?

   $\boxed{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$

2. $(2x-1)^3$ $\binom{3}{0}(2x)^3 + \binom{3}{1}(2x)^2(-1) + \binom{3}{2}(2x)(-1)^2 + \binom{3}{3}(-1)^3$

   $n=3$ $(1)(2)^3x^3 + (3)(2)^2x^2(-1) + (3)(2)x(1) + (1)(-1)$

   $a=2x$ $8x^3$ $-3(4)x^2$ $6x$ -1

   $b=-1$

   $\boxed{8x^3 - 12x^2 + 6x - 1}$

Assignment 4.5

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Day 6

Unit 4 Review

Assignment 4.6   ***All late/absent assignments due for Unit 4***

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Day 7

Unit 4 Test

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