

Unit 6 Notes

Trigonometry *triangle measurement*

Day 1

Pronounced "theta"

Right Triangle Trigonometry

Trigonometry means "measurement of trigons" (triangles). The easiest angles to deal with in trigonometry are the acute angles in right triangles, so we will start there. In the right triangle below, one acute angle is named θ , and with respect to that angle, you need to be able to identify the *opposite side*, the *adjacent side*, and the *hypotenuse* of the triangle.

Primary Functions:

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \text{S O H}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \text{C A H}$$

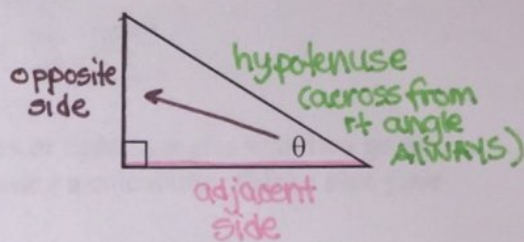
$$\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \text{T O A}$$

Reciprocal Functions:

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

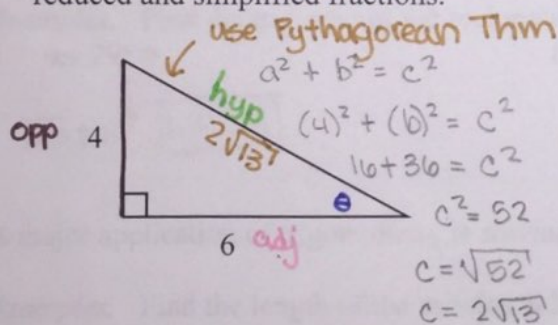


SOH-CAH-TOA

Example.

- Use the right triangle below to find the **exact values** (trig ratios) of the six trigonometric functions of θ . You will need to find the third side using the Pythagorean Theorem. **Exact values** means no decimals; reduced and simplified fractions.

More Room!



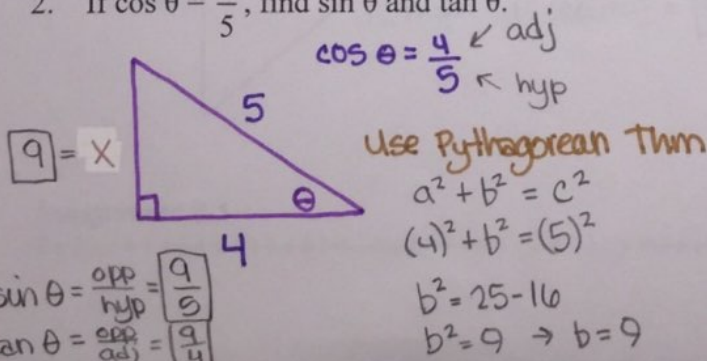
NO decimals!

$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{4}{2\sqrt{13}} = \frac{4\sqrt{13}}{2(13)} = \frac{2\sqrt{13}}{13} & \csc \theta &= \frac{2\sqrt{13}}{4} = \frac{\sqrt{13}}{2} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{6}{2\sqrt{13}} = \frac{3\sqrt{13}}{13} & \sec \theta &= \frac{2\sqrt{13}}{6} = \frac{\sqrt{13}}{3} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} = \frac{4}{6} = \frac{2}{3} & \cot \theta &= \frac{3}{2} \end{aligned}$$

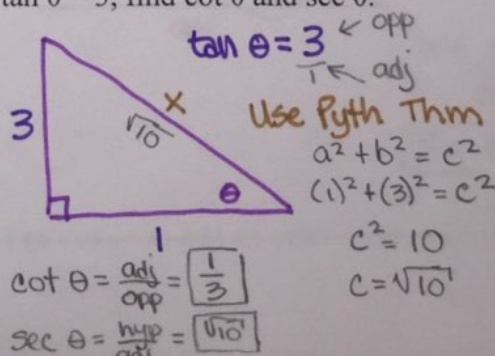
If you know a trigonometric ratio, you know two sides of a right triangle. Draw the triangle, label the sides, find the third side and then the other ratios. **Must be a right triangle.**

Examples.

- If $\cos \theta = \frac{4}{5}$, find $\sin \theta$ and $\tan \theta$.

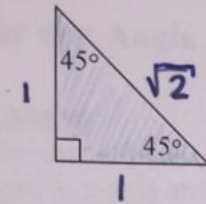


- If $\tan \theta = 3$, find $\cot \theta$ and $\sec \theta$.

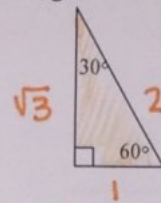


There are two "special" right triangles. You should memorize the side lengths to help you build trig ratios:

45° - 45° - 90° :



30° - 60° - 90° :



Example.

4. Find the **exact** value of the following: S O C H C A H T A

$$\sin 45^\circ = \frac{O}{H} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin 30^\circ = \frac{O}{H} = \frac{1}{2}$$

$$\sin 60^\circ = \frac{O}{H} = \frac{\sqrt{3}}{2}$$

$$\cos 45^\circ = \frac{A}{H} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 30^\circ = \frac{A}{H} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{A}{H} = \frac{1}{2}$$

$$\tan 45^\circ = \frac{O}{A} = \frac{1}{1} = 1$$

$$\tan 30^\circ = \frac{O}{A} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan 60^\circ = \frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

It is possible to find exact values of trig ratios when using special triangles or right triangles with two given sides. However, you may need to find approximate values of trig ratios using a calculator. (Make sure your calculator is in the correct mode.) **Degree Mode**

Examples. Find the trig ratio using your calculator. Round to four decimal places.

5. $\sin 38^\circ \approx 0.6157$

6. $\tan 82^\circ \approx 7.1154$

Sines, cosines and tangents are easy to find in the calculator because there is a key for each. But the secondary functions (secant, cosecant, and cotangent) are found using the reciprocal of the function: $1/\cos(\text{angle})$, $1/\sin(\text{angle})$ or $1/\tan(\text{angle})$.

Examples. Find the trig ratio of the reciprocal functions (rounded to four decimal places):

7. $\sec 29^\circ \approx$

8. $\csc 48^\circ \approx$

$$\frac{1}{\cos 29} = 1.1434$$

$$\frac{1}{\sin 48} = 1.3456$$

A major application of trigonometry is solving for some (or all) of the missing parts of a right triangle.

Examples. Find the length of the missing side (x) using an equation involving a trig ratio. Round answers to the nearest tenth.

9. $C A H \rightarrow$
 $\cos 41^\circ = \frac{x}{12}$
 $12(\cos 41) = 9.1$

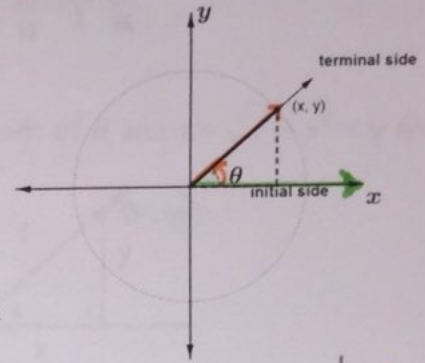
10. $S O H$
 $\sin 22^\circ = \frac{6}{x}$
 $x(\sin 22) = 6$
 $x = \frac{6}{\sin 22}$
 $x = 16.0$

Assignment 8.1

Trigonometry for any Angle

Standard Position Angles

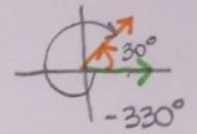
Let the origin of a coordinate plane be the **vertex** of an **angle** whose **initial side** is the positive x-axis and whose **terminal side** forms an angle measuring θ with respect to the initial side.



Positive Angles = counterclockwise ↺ from initial side

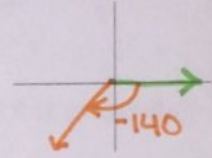
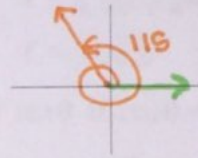
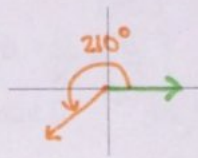
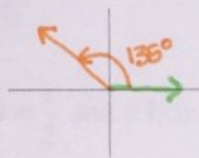
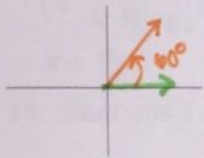
Negative Angles = clockwise ↻ from initial side

Coterminal angles = angles that share same initial side & terminal side (CCW & CW)



Examples: Draw the following angles in standard position.

1. 60°
2. 135°
3. 210°
4. 475°
5. -140°

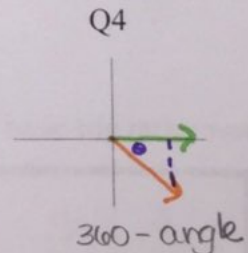
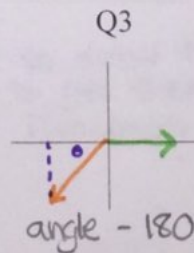
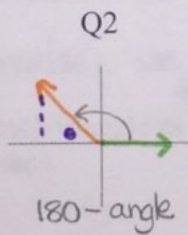
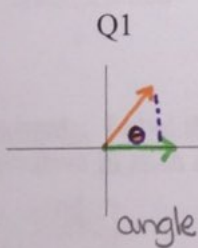


Find one positive and one negative angle that is coterminal with the given angle.

6. 120°
 $120^\circ + 360^\circ = 480^\circ$ pos
 $120^\circ - 360^\circ = -240^\circ$ neg
7. -420°
 $-420^\circ + 360^\circ = -60^\circ$ neg
 $-420^\circ + 2(360^\circ) = 300^\circ$ pos

Reference Angle:

The acute angle between the terminal side and the x-axis. (between 0° and 90°)
 Always positive.



Examples: Find the measure of the reference angle for the given angles.

8. 120°
 $\theta = 180 - 120 = 60^\circ$
9. 30°
 $\theta = 30^\circ$
10. 245°
 $\theta = 245 - 180 = 65^\circ$
11. 342°
 $\theta = 360 - 342 = 18^\circ$
12. -115°
 $\theta = 180 - 115 = 65^\circ$

Reference Triangle:

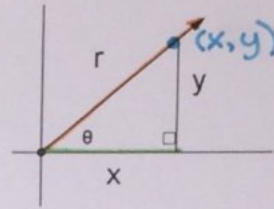
Right triangle containing the reference angle and the x-axis.
Used to find trig ratios for angles in standard position.

S O C H A T A

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$ as shown in the figure. Then:

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}$$



Examples:

Find sin, cos, and tan if the terminal side of θ goes through the given point.

13. $(3, 4)$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(3)^2 + (4)^2}$$

$$r = \sqrt{9 + 16} = \sqrt{25}$$

$$r = 5$$

$$x = 3$$

$$y = 4$$

$$r = 5$$

$$\sin \theta = \frac{y}{r} = \frac{4}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{4}{3}$$

14. $(-4, 8)$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-4)^2 + (8)^2}$$

$$r = \sqrt{16 + 64} = \sqrt{80}$$

$$r = 4\sqrt{5}$$

$$x = -4$$

$$y = 8$$

$$r = 4\sqrt{5}$$

$$\sin \theta = \frac{8}{4\sqrt{5}}$$

$$\sin \theta = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{-4}{4\sqrt{5}}$$

$$\cos \theta = \frac{-\sqrt{5}}{5}$$

15. Find $\cos \theta$ if $\sin \theta = \frac{1}{2}$ and θ is in Quadrant II.

More room

$$\sin \theta = \frac{1}{2} \rightarrow y = 1$$

$$r = 2$$

$$x = -\sqrt{3}$$

$$\cos \theta = \frac{x}{r}$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$

16. Find $\tan \theta$ if $\cos \theta = \frac{2}{5}$ and θ is in Quadrant IV.

$$\cos \theta = \frac{2}{5} \rightarrow x = 2$$

$$r = 5$$

$$y = -\sqrt{21}$$

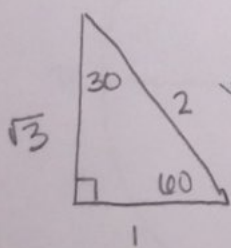
$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = -2$$

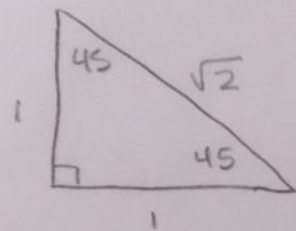
$$r^2 = x^2 + y^2 \rightarrow x = \sqrt{r^2 - y^2} = \sqrt{4 - 1} = \pm\sqrt{3}$$

$$r^2 = x^2 + y^2 \rightarrow y = \sqrt{r^2 - x^2} = \sqrt{25 - 4} = \sqrt{21}$$

Trig Ratios: Find the following ratios using the "special" triangles. These need to be MEMORIZED.

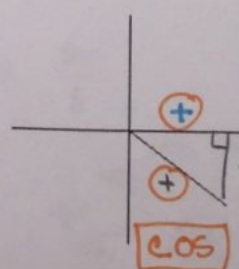
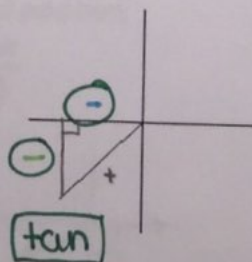
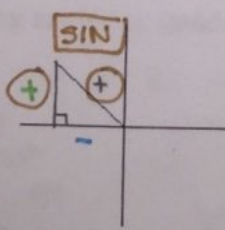
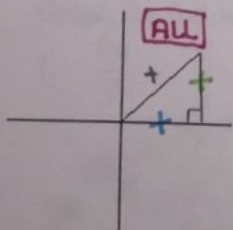


	30°	45°	60°
sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan θ	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



Memorize: or be able to draw the triangles to get them.

Quadrants: Label the sides of each triangle as "+" or "-". Then identify which of the basic trig ratios would be positive in each of the quadrants.

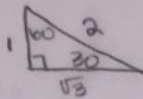
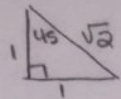


S	A
T	C

MEMORIZE!!!

All Students Take Care

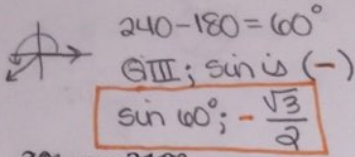
S	A
T	C



S O C H T A

Examples: Evaluate the following without using a calculator. (HINT: use quadrant and reference angle)

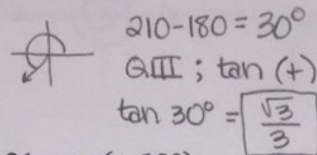
17. $\sin 240^\circ$ reference angle



2011 sec 210°

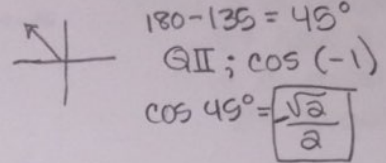
Do not include next year

18. $\tan 210^\circ$



2111 cot (+60°)

19. $\cos 135^\circ$



Assignment 6.2

Day 3

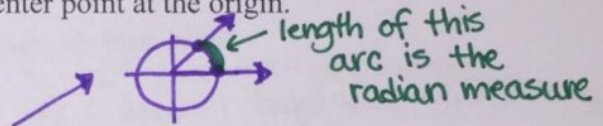
Radians

The unit circle is a circle with a radius of one unit and a center point at the origin.

Radians:

Different way of measuring angles.

The length of the arc of the unit circle that is inside the angle you are measuring.



What is the circumference of the unit circle?

$C = 2\pi r$, so $C = 2\pi(1) = 2\pi$
 So, $360^\circ = 2\pi$ radians

What is the distance halfway around? $\frac{2\pi}{2} = \pi$

* So, $180^\circ = \pi$

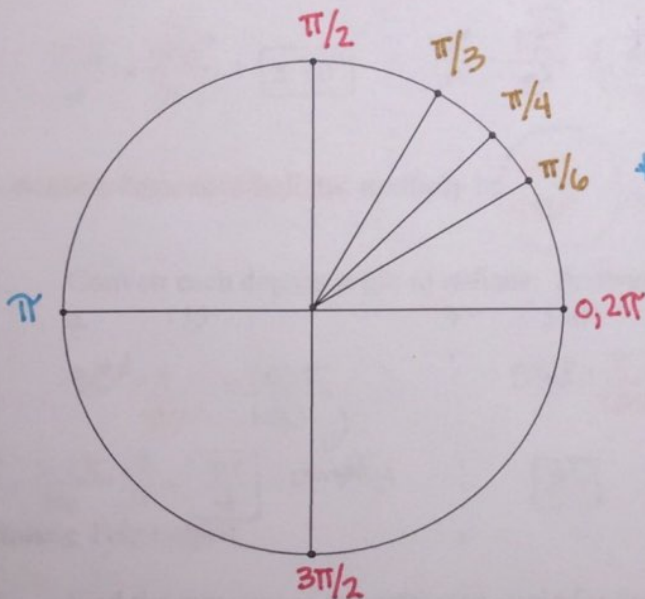
$90^\circ = \frac{\pi}{2}$

$270^\circ = \frac{3\pi}{2}$

$30^\circ = \frac{\pi}{6}$ ($\frac{180}{6} = 30^\circ$)

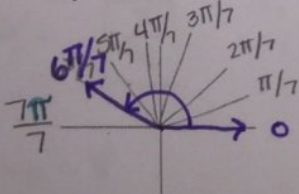
$45^\circ = \frac{\pi}{4}$ ($\frac{180}{4} = 45^\circ$)

$60^\circ = \frac{\pi}{3}$ ($\frac{180}{3} = 60^\circ$)

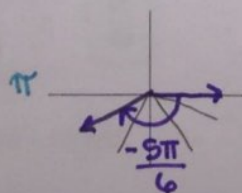


Graph the following angles in standard position.

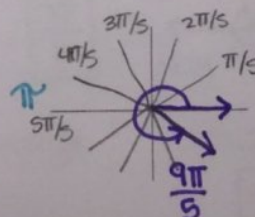
1. $\frac{6\pi}{7}$



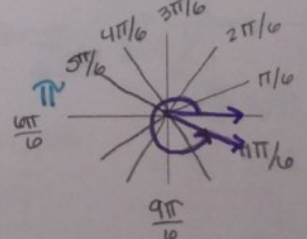
2. $-\frac{5\pi}{6}$

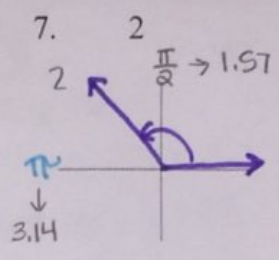
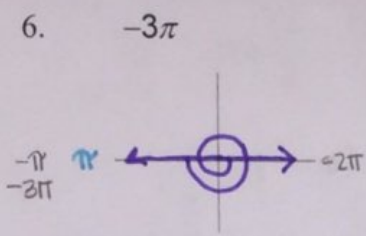
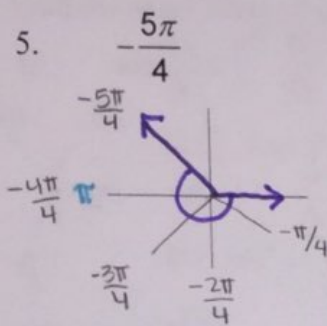


3. $\frac{9\pi}{5}$



4. $\frac{11\pi}{6}$





$\pi = 3.14$
 $\frac{\pi}{2} = 1.57$
 Since 2 is between 1.57 and 3.14 it is somewhere in 2nd quadrant

Find one positive angle and one negative angle that are coterminal to the given angle.

8. $\frac{\pi}{3}$

9. $-\frac{3\pi}{4}$

pos: $\frac{\pi}{3} + 2\pi = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$
 neg: $\frac{\pi}{3} - 2\pi = \frac{\pi}{3} - \frac{6\pi}{3} = \frac{-5\pi}{3}$

pos: $-\frac{3\pi}{4} + 2\pi = -\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$
 neg: $-\frac{3\pi}{4} - 2\pi = -\frac{3\pi}{4} - \frac{8\pi}{4} = \frac{-11\pi}{4}$

Converting Units:

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi}$. This is equivalent to 1 (and when you multiply by 1 you don't change value → just the way it looks)

10. Convert each radian angle to degrees. On d & e, round to 2 decimal places.

a. $\frac{13\pi}{6} \cdot \frac{180^\circ}{\pi} = 390^\circ$

b. $\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = 135^\circ$

c. $\frac{-2\pi}{3} \cdot \frac{180^\circ}{\pi} = -120^\circ$

d. $5.62 \cdot \frac{180^\circ}{\pi} = 322.00^\circ$
 e. $-1.63 \cdot \frac{180^\circ}{\pi} = -93.39^\circ$

To convert degrees to radians, multiply by $\frac{\pi}{180^\circ}$. This is equivalent to 1

11. Convert each degree angle to radians. Answer in exact form.... No decimals.

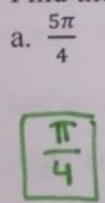
a. $135^\circ \cdot \frac{\pi}{180^\circ} = \frac{135\pi}{180}$
 $\frac{135\pi}{180} = \frac{27\pi}{36} \div 9 = \frac{3\pi}{4}$ simplify

b. $540^\circ \cdot \frac{\pi}{180^\circ} = \frac{540\pi}{180} = 3\pi$

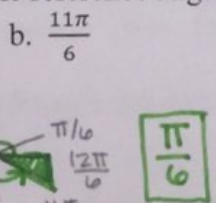
c. $-270^\circ \cdot \frac{\pi}{180^\circ} = \frac{-270\pi}{180} = -\frac{3\pi}{2}$

Finding Trig ratios:

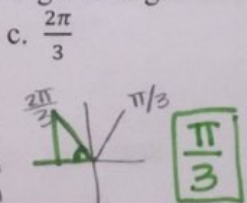
12. Find the measure of the reference angle for the given angles.



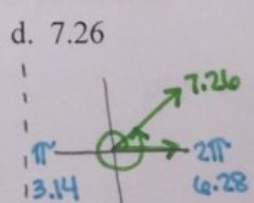
Q3: angle $-\pi$
 $\frac{5\pi}{4} - \pi = \frac{5\pi}{4} - \frac{4\pi}{4} = \frac{\pi}{4}$



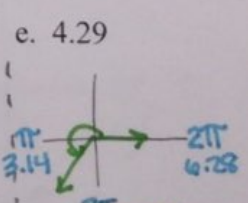
Q: 4: 2π -angle



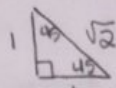
Q2: π -angle



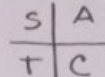
$7.26 - 6.28 = 0.98$



$4.29 - 3.14 = 1.15$



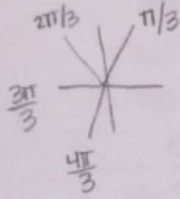
SIN CAT A



Find the exact value of the following. Remember quadrant and reference angles?

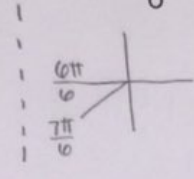
13. $\sin \frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$

Q III (sin neg)
ref L = $\frac{\pi}{3}$ (60°)
 $\sin 60^\circ = \frac{\sqrt{3}}{2}$



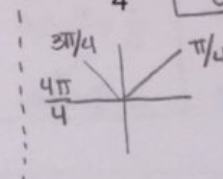
14. $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

Q III (tan pos)
ref L = $\frac{\pi}{6}$ (30°)
 $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$



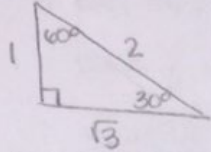
15. $\cos \frac{3\pi}{4} = \frac{-\sqrt{2}}{2}$

Q II (cos neg)
ref L = $\frac{\pi}{4}$ (45°)
 $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



16. $\sec \frac{7\pi}{6}$

17. $\cot \left(\frac{11\pi}{3} \right)$

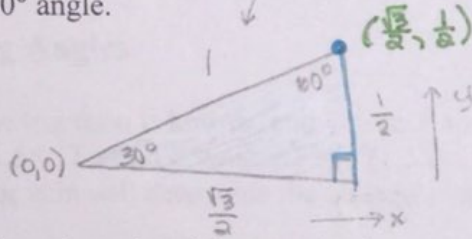


Assignment 6.3

Day 4

Axis Angles / Unit Circle

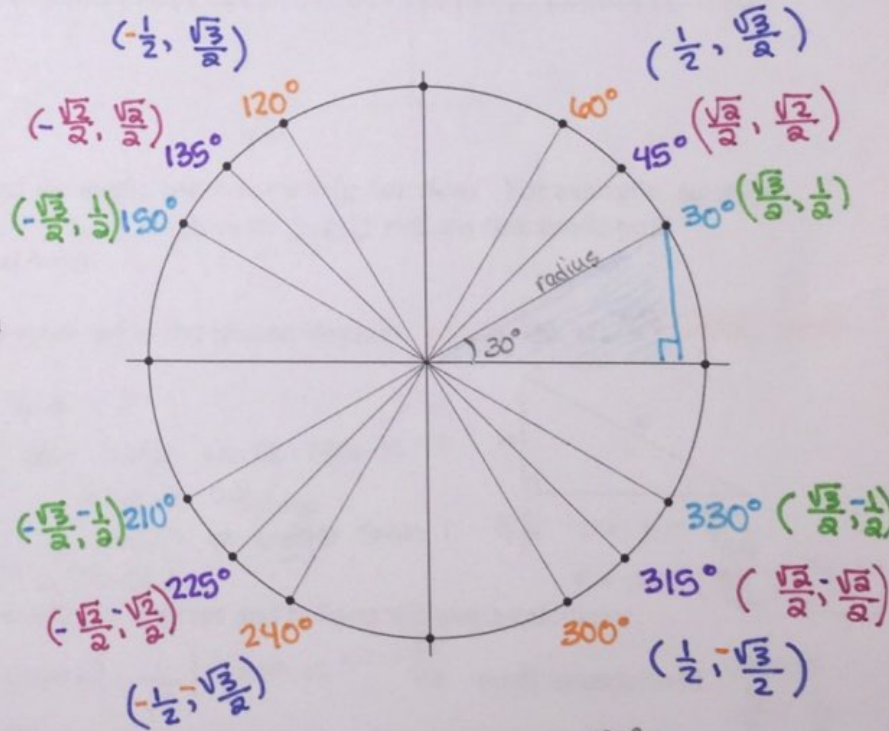
Find the point of intersection for the unit circle and a 30° angle.



What does the point of intersection between an angle and the unit circle tell you?

$x = \cos \theta$
 $y = \sin \theta$

if (x, y) is a point on the unit circle.

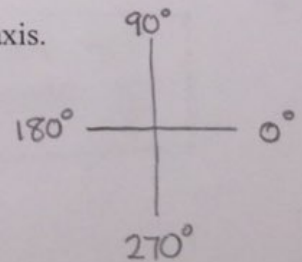


Axis angles:

Axis angles are angles in standard position whose terminal side lies on a coordinate axis. The 4 primary axis angles measure $0^\circ, 90^\circ, 180^\circ,$ and 270° .

**How can we find trig ratios for axis angles?

* Use the point on the unit circle because we can't draw a reference triangle.



$$\cos \theta = x \quad \sin \theta = y$$

Find sin, cos, and tan for each given angle without using a calculator.

1. $\theta = 0$

$$\sin 0 = y \text{ value @ } 0^\circ = 0$$

$$\cos 0 = x \text{ value @ } 0^\circ = 1$$

$$\tan 0 = \frac{y}{x} = \frac{0}{1} = 0$$

3. $\theta = \pi$

$$\begin{aligned} \sin \pi &= 0 \\ \cos \pi &= -1 \\ \tan \pi &= \frac{0}{-1} = 0 \end{aligned}$$

5. Find the following ratios without a calculator.

a. $\sin 180^\circ$

$$y \text{ value @ } 180^\circ$$

$$\boxed{0}$$

b. $\tan(-90^\circ)$

$$\tan = \frac{y}{x} = \frac{-1}{0}$$

$$\boxed{\text{UNDef}}$$

c. $\sec 90^\circ$

$$\rightarrow \cos$$

$$\cos 90^\circ = x = 0$$

$$\sec 90^\circ = \frac{1}{0}$$

$$\boxed{\text{UNDef}}$$

d. $\csc 270^\circ$

$$\cot \rightarrow \tan$$

$$\rightarrow \sin$$

$$\sin 270^\circ = y = -1$$

$$\csc 270^\circ = \frac{1}{-1} = \boxed{-1}$$

Remember:

The reciprocal of 0 is undefined

$\frac{1}{0}$ \leftarrow b/c can't have zero in denominator

The reciprocal of undefined is zero

Assignment 6.4

Day 5

Finding Angles

When the trig ratio is known, and you need to find the angle, use *inverse trig functions*. For example, suppose $\tan \theta = 1.56$. To find θ , evaluate $\tan^{-1}(1.56)$: $\theta \approx 57.3$ degrees or 1.00 radians (the mode your calculator is in will determine the units of your answer).

Example. Use a calculator to find θ or x in degrees (round to the nearest degree). **Must be in DEGREE mode**

1. $\sin \theta = 0.7695$

$$\theta = \sin^{-1}(0.7695)$$

$$\boxed{\theta = 50^\circ}$$

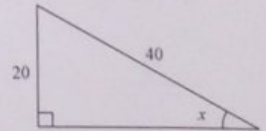
2. $\sec \theta = -3$

$$\rightarrow \cos \text{ angle whose cos is } -3$$

* cos is adj/hyp
can't be bigger than 1

$$\boxed{\text{UNDEFINED}}$$

3.



$$\sin x = \frac{20}{40}$$

$$x = \sin^{-1}\left(\frac{20}{40}\right) = \boxed{30^\circ}$$

Example. Find the smallest positive value of θ exactly in degrees and radians without a calculator.

4. $\sin \theta = \frac{\sqrt{3}}{2}$ when is $y = \frac{\sqrt{3}}{2}$

$$\text{Degrees: } 60^\circ$$

$$\text{Radians: } \frac{\pi}{3}$$

5. $\cos \theta = \left(-\frac{1}{\sqrt{2}}\right)$ when is $x = -\frac{\sqrt{2}}{2}$. $\tan \theta$ is undefined when is $\frac{y}{x} = \frac{1}{0}$

$$\text{Degrees: } 135^\circ$$

$$\text{Radians: } \frac{3\pi}{4}$$

$$\text{Degrees: } 90^\circ$$

$$\text{Radians: } \frac{\pi}{2}$$

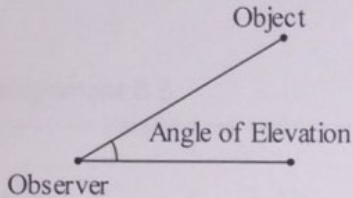
Convert from degrees to radians
multiply by $\frac{\pi}{180}$

Applications involving Trigonometry

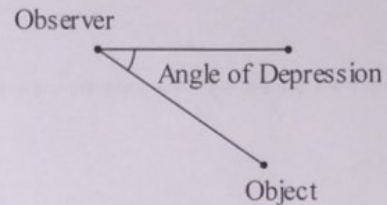
Many real world applications can be solved using trigonometry. You will need to draw a picture (right triangle) that models the situation, label the picture, and write a trig equation to solve.

It's important to know the definition of two descriptions of angles often used in describing these situations:

An **angle of elevation** is an angle measured from the horizontal upward toward an object.

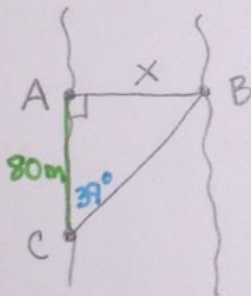


An **angle of depression** is an angle measured from the horizontal downward toward an object.



Examples.

7. To measure the width of a river, you plant a stake at point A on one side of a riverbank, directly across from a tree stump at point B on the other side of the river. From point A, you walk 80 meters along the riverbank to point C. You measure $\angle ACB$ and find it to be 39° . How wide is the river?



S_H^O C_H^A T_A^O use 39° as your angle

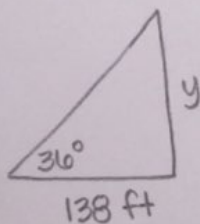
$$S_H^O \rightarrow \sin 39^\circ = \frac{x}{80} \quad (\text{multiply both sides by 80 to get it out of the denominator})$$

$$80(\sin 39^\circ) = x \quad (\text{plug } 80(\sin 39^\circ) \text{ in calculator})$$

⚡ be sure you have switched back to degree mode

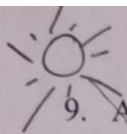
$$x = 50.3 \text{ m}$$

8. The Falls Incline Railway at Niagara Falls has an angle of elevation of 36° . The railway extends a horizontal distance of about 138 feet. Find the height and length of the railway.

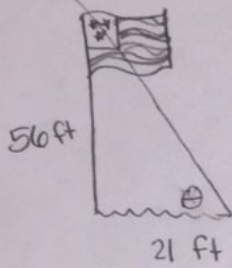


$$\sin 36^\circ = \frac{y}{138} \rightarrow y = 138(\sin 36)$$

$$y = 81.1 \text{ ft}$$



9. A flagpole that is 56 feet tall casts a shadow 21 feet long. Find the angle of elevation of the sun from the tip of the shadow.



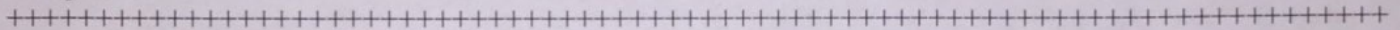
$$\tan \theta = \frac{56}{21}$$

$$\theta = \tan^{-1} \left(\frac{56}{21} \right)$$

$$\theta = 69.4^\circ$$

* When need to find angle
use \sin^{-1} \cos^{-1} \tan^{-1}

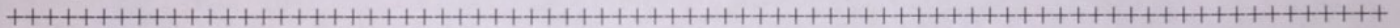
Assignment 6.5



Day 6

Unit 6 Review

Assignment 6.6



Day 7

Unit 6 Test

All late/absent assignments due for Unit 6

