Unit 1
Functions and Polynomials

Day 1
Comparing Multiple Representations of a Function

Function vocabulary:

Relation Set of ordered pairs (points) (x, y)

Function a relation where each input value is used exactly once (no repeats of x values)

Domain all input values (x)

Range all output values (y)

Function families:

- **Linear** rate of change is constant: can be written $y = mx + b$
- **Quadratic** input is squared: $y = ax^2 + bx + c$
- **Exponential** input is a power: $y = a \cdot b^x$

Ways to represent functions
- **Graphs**
- **Tables**
- **Equations**

Example 1:
List the domain and range for the following relations. Determine if the table of values represents a function. If so, determine the function family with which it is associated.

1.

x	y
−5	−13
−4	−12
0	−8
1	−7
5	−3

Domain: $\{-5, -4, 0, 1, 5\}$ Range: $\{-13, -12, -8, -7, -3\}$
Function? YES / NO
Function Family: Linear / Quadratic / Exponential

2.

x	y
−2	−2
0	0
1	4
3	18
5	40

Domain: $\{-2, 0, 1, 3, 5\}$ Range: $\{-2, 0, 4, 18, 40\}$
Function? YES / NO
Linear / Quadratic / Exponential
? Not enough info

3.

x	y
4	1
4	2
4	3
4	4
4	5

Domain: $\{4\}$ Range: $\{1, 2, 3, 4, 5\}$
Function? YES / NO
Linear / Quadratic / Exponential
vertical line

Function notation: $f(x)$ input variable
name of function

Using function notation to evaluate functions:

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Example 2: Given the relation $y = 4x - 7$,

a. Is this relation a function? If it is, write it using function notation with the name f

Yes $f(x) = 4x - 7$

b. Find $f(3) = 4(3) - 7 = 12 - 7 = \boxed{5}$

c. Find $f(-2) = 4(-2) - 7 = -8 - 7 = \boxed{-15}$

d. Find $f(x+3) = 4(x+3) - 7 = 4x + 12 - 7 = \boxed{4x+5}$

d. What is the difference between $f(3)$ and $f(x+3)$?

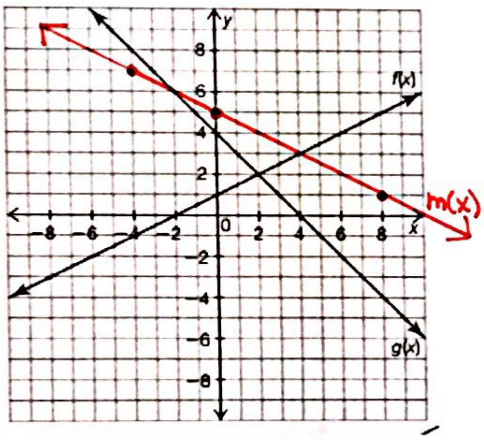
if you plug in a # you will get a # answer.

an expression will give a new function

Analyzing Graphs to Build New Functions

The graph of a function is the set of an infinite number of points. When two functions are added together, the output values for each input value are added together. You can graphically add these together to get a graph for the new function by considering key points (intercepts, zeros and intersection points). Be sure to label the new graph.

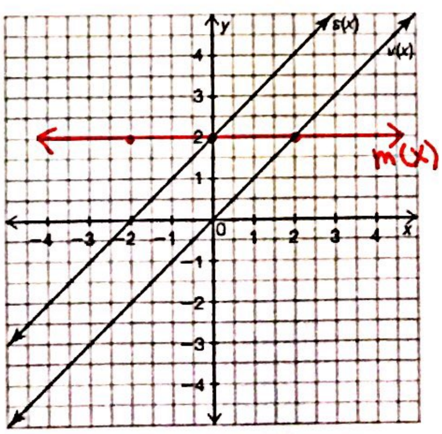
Example 3: Analyze the graph of $f(x)$ and $g(x)$ and use key points to graph $m(x) = f(x) + g(x)$.



x	$f(x)$	$g(x)$	$m(x)$
-4	-1	8	7
0	1	4	5
8	5	-4	1

Graph m(x) $\begin{cases} (-4, 7) \\ (0, 5) \\ (8, 1) \end{cases}$

Example 4: The same idea works for subtraction as well. Analyze the graphs of $s(x)$ and $v(x)$. Use key points to draw the graph of $m(x) = s(x) - v(x)$



x	$s(x)$	$v(x)$	$m(x)$
0	2	0	2
-2	0	-2	2
2	4	2	2

Graph m(x) $\begin{cases} (0, 2) \\ (-2, 2) \\ (2, 2) \end{cases}$

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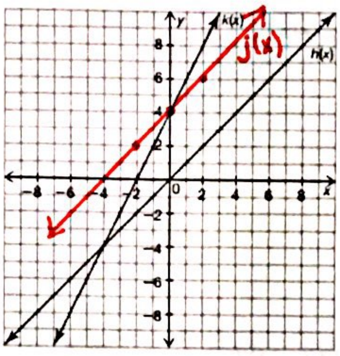
Example 5: Now try it backwards. Analyze the graphs of $h(x)$ and $k(x)$ and graph the function $j(x)$ so that $h(x)+j(x)=k(x)$.

Graph j(x) (0,4) (-2,2) (2,6)



Given ↓ Given ↓

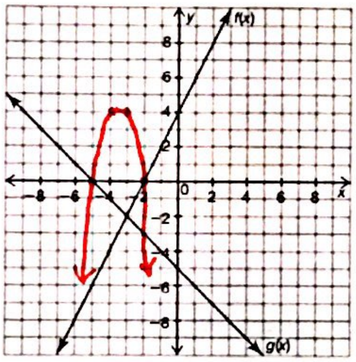
x	$h(x)$	$j(x)$	$k(x)$
0	0	4	4
-2	-2	2	0
2	2	6	8

0 + ? = 4
-2 + ? = 0
2 + ? = 8

Adding or subtracting two linear functions creates a new function that is also linear. What happens when two linear functions are multiplied?

Example 6: Analyze the graphs of $f(x)$ and $g(x)$. Predict the function family of $h(x)$ if $h(x)=f(x)\cdot g(x)$. Then draw the graph of $h(x)$.

Graph h(x) (-3,4) (-4,4) (-2,0) (-5,0)



Function family: Quadratic

x	$f(x)$	$g(x)$	$h(x)$
0	4	-5	-20
-2	0	-3	0
-5	-6	0	0
-4	-4	-1	4
-3	-2	-2	4

Assignment 1.1

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Day 2

Polynomial Algebra

Polynomial vocabulary:

- Term number and/or variable : $2x$, $\frac{n}{3}$, $5w$, etc.
- Coefficient number multiplying a variable
- Degree largest power of variable
- Leading Coefficient Coefficient of variable raised to largest power
- Constant number not multiplying a variable
- Monomial, Binomial, Trinomial Polynomials classified by number of terms
  1 term 2 terms 3 terms
- Factor numbers or expressions we multiply together to get a larger number or expression.

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Operations with Polynomials (add, subtract, multiply):

How do you add terms of a polynomial?

Combine like terms (same variable and power)

How do you multiply polynomials by other polynomials?

Multiply coefficients and add powers

Examples. Add, subtract or multiply as indicated:

1. $(3a^5-9a^3+4a^2)+(-8a^5+8a^3+2)$

$-5a^5-a^3+4a^2+2$

2. $(-6m^2-8m+5)-(-5m^2+7m-8)$

$-m^2-15m+13$

3. $5x^2(-4x^2+3x-2)$

$-20x^4+15x^3-10x^2$

4. $(4t-5)(3t+1)$

$12t^2+4t-15t-5$

$12t^2-11t-5$

5. $(3-5x)^2$ $(3-5x)$ FOIL

$9-15x-15x+25x^2$

~~equals~~ $9-30x+25x^2$

Standard Form: $25x^2-30x+9$

6. $2(5a+3)^2-1$ order of operations

$2(5a+3)(5a+3)-1$

$2(25a^2+15a+15a+9)-1$

$2(25a^2+30a+9)-1$

$50a^2+60a+18-1$

$\boxed{50a^2+60a+17}$

Combining linear and/or quadratic functions:

Examples. Given $f(x)=2x+1$ and $g(x)=x^2-2$, find the following:

7. $f(x)+g(x)$

$2x+1 \quad + \quad x^2-2 \quad = \boxed{x^2+2x-1}$

8. $f(x)-g(x)$

$2x+1 \quad -(x^2-2) \quad = \boxed{-x^2+2x+3}$

$2x+1-x^2+2$

9. $f(x)\cdot g(x)$

$(2x+1)(x^2-2)$

$2x^3-4x+x^2-2$

$\boxed{2x^3+x^2-4x-2}$

10. $f(-2)+g(-2)$

$2(-2)+1 \quad +(-2)^2-2$

$-4+1+4-2$

$\boxed{-1}$

Factoring

What are the factors of 10?

$1\cdot 10 \qquad 2\cdot 5$

What are the factors of $3x^2$?

$X \cdot X$

What is does it mean to factor a polynomial?

Break it down in to smaller terms

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Factor out Greatest Common Factor (GCF) Find largest factor & divide it out of each term.

Examples. Factor out the greatest common factor:

11. $5z+5$

$5(z+1)$

12. $9x^2-12x^3$

$3x^2(3-4x)$

13. $24k^5+16k^4-8k^2$

$8k^2(3k^3+2k^2-1)$

14. $24m^3n^2-18m^2n+6m^2n$

$6m^2n(4mn-3+1)$

$6m^2n(4mn-2)$

Factoring out a negative

Sometimes it's helpful to factor out -1 so that the first term in a polynomial is positive.

Examples: Factor out -1 to change the signs on all terms.

15. $-x^2+3x-27$

$-(x^2-3x+27)$

16. $-2t+3$

$-(2t-3)$

Assignment 1.2

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Day 3

Factoring Trinomials

Multiply $(a+2)(a-4)$: $a^2-4a+2a-8 = a^2-2a-8$

The factors of a^2-2a-8 are: $(a+2)(a-4)$

Factor trinomials of the form x^2+bx+c

- **Factor out common factors FIRST** (this includes factoring out a -1 if the quadratic term is negative).
- Factors into two binomials (if the polynomial is not prime)
- The first term and the last term are products.
- The second term is a product AND a sum.
- Look for two numbers whose product = c and whose sum = b.

Examples. Factor the trinomials:

1. $x^2-2x-15$

$(x-5)(x+3)$

2. $a^2-7a+10$

$(a-5)(a-2)$

3. b^2+3b-5 Doesn't Factor

$(b\quad)(b\quad)$

Prime

4. $p^2+6p-16$

$(p+8)(p-2)$

5. $-2x^2+12x-16$

$-2(x^2-6x+8)$

$-2(x-4)(x-2)$

6. $16y^3-32y^2-48y$

$16y(y^2-2y-3)$

$16y(y-3)(y+1)$

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Factor trinomials of the form $ax^2 + bx + c$

- Remember to factor out greatest common factor FIRST.
- These can be factored by trial and error (erasers are helpful!)
- Determine the factors of a and the factors of c. Try different combinations until the middle term = bx when the factors are multiplied together.

Examples. Factor the trinomial completely:

7. $3x^2+7x+2$

$(3x+1)(x+2)$

8. $2k^2-5k-3$

$(2K+1)(K-3)$

9. $6p^2-19p+10$

$(3p-2)(2p-5)$

10. $6a^2+13a-5$

$(3a-1)(2a+5)$

11. $-16y^3-24y^2+16y$

$-8y(2y^2+3y-2)$

$-8y(2y-1)(y+2)$

Factor special polynomials

Multiply the binomials: $(x+5)(x-5)$ $x^2-5x+5x-25 = x^2-25$

Difference of Squares notice: no middle term

Formula: $a^2-b^2=(a+b)(a-b)$

Sum of Squares

What about a^2+b^2 ? Does it factor? Why or why not? NO! There is no way for the middle term to cross out and be positive.

Examples. Factor the polynomials completely:

12. y^2-49

$(y+7)(y-7)$

13. $1-16y^2$

$(1-4y)(1+4y)$

13. $4x^2-1$

$(2x-1)(2x+1)$

14. $9a^2-25b^2$

$(3a-5b)(3a+5b)$

15. $27x^2-48y^2$

$3(9x^2-16y^2)$

$3(3x-4y)(3x+4y)$

16. x^2+64

Prime

Assignment 1.3

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Day 4

Solving Equations and Inequalities

What does it mean to solve an equation? Find the values of x that make it true.

Solve linear equations

- Simplify each side first (distribute and combine like terms).
- Add/subtract from BOTH SIDES so that the variable is on one side and a number on the other.
- Multiply/divide on BOTH SIDES so that the variable is alone.

Examples. Solve the following equations for x. Check answers:

1. $3x+2=3(2x-5)$

$3x+2=6x-15$
$-3x$
$2=3x-15$
$+15$
$17=3x$
$\boxed{x=\frac{17}{3}}$

2. $6\left(\frac{8x}{3}-\frac{1}{2}x\right)=(-13)6$

$\cancel{6}^{2}\cdot\frac{8x}{\cancel{3}}-\cancel{6}^{3}\cdot\frac{1}{\cancel{2}}x=-78$

$16x-3x=-78$

$13x=-78$

$\boxed{x=-6}$

3. $-[2x-(5x+2)]=2x+9$

$-[2x-5x-2]=2x+9$

$-2x+5x+2=2x+9$

$3x+\cancel{2}=2x+9$
-2

$3x=2x+7$
$-2x$

$\boxed{x=7}$

4. $0.05x+0.12(x+5000)=940$

$.05x+.12x+600=940$

$.17x+600=940$
-600

$.17x=340$

$\boxed{x=2000}$

Solving polynomial equations by factoring

- Uses the zero property of multiplication:

  any number • 0 = 0

- Set equation = 0
- Factor
- Set each factor = 0 and find *x*.

Examples – Use factoring to solve each quadratic equation.

4. $3x^2+x=0$

$x(3x+1)=0$

$\boxed{x=0}$ $3x+1=0$

$3x=-1$

$\boxed{x=-1/3}$

5. $x^2-4x+4=0$

$(x-2)(x-2)=0$

$x-2=0$

$\boxed{x=2}$

6. $p^2-12p=-32$

$p^2-12p+32=0$

$(p-8)(p-4)=0$

$\boxed{p=8 \quad p=4}$

7. $(2k+1)(k+1)=8-2k$

$2k^2+2k+k+1=8-2k$

$2k^2+3k+1=\cancel{8}-\cancel{2k}$
$+2k \quad -8$

$2k^2+5k-7=0$

$7 \quad (2k+7)(k-1)=0$

$2k+7=0 \quad k-1=0$

$\boxed{k=-7/2 \quad k=1}$

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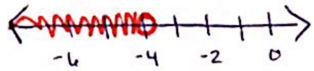
Solving linear inequalities

Express the set of real numbers that are less than −4 in three ways:

As a graph on the number line: 

As an inequality: $x < -4$

Using interval notation: $(-\infty, -4)$

The process of solving linear inequalities is similar to solving equations: get the variable alone on the left side of the inequality. The answers are infinite and can be represented by a graph on the number line or using inequality notation.

BE CAREFUL when you multiply or divide both sides of the inequality by a negative quantity:

the sign flips!

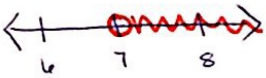
Examples. Solve the inequality. Express the result with a graph on the number line and with interval notation:

8. $2x+3>17$

$-3 \quad -3$

$2x > 14$

$x > 7 \qquad (7, \infty)$

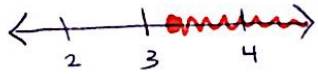


9 $2x+3 \le 5x-7$

$-5x \qquad -3$

$\frac{-3x}{-3} \le \frac{-10}{-3}$ ← sign flips!

$x \ge \frac{10}{3} \qquad [\frac{10}{3}, \infty)$



10. $6+2y<2(y+4)$

$6+2y < 2y+8$

$-2y \quad -2y$

$6 < 8$

↑

This is a true statement so any y value will work.

(∞, ∞) 

11. $2(x-1) \le 3-2(x+3)$

$2x-2 \le 3-2x-6$

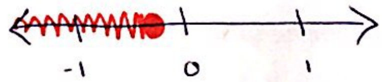
$+2x$

$4x-2 \le -3$

$+2$

$4x \le -1$

$x \le -1/4 \qquad (-\infty, -1/4]$



Assignment 1.4

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Day 5

Unit 1 Review

Assignment 1.5

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Day 6

Unit 1 Test ***All late/absent assignments due for Unit 1***

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