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Unit 2 Notes / Secondary 3 Honors

Day 1: Quadratic Functions

- QUADRATIC FUNCTION STANDARD FORM: $f(x) = ax^2 + bx + c$ *a, b, & c are real #s*

  (0, c) is y-intercept

- FACTORED FORM OF A QUADRATIC: $f(x) = a(x - r_1)(x - r_2)$ $(r_1, 0)$ and $(r_2, 0)$ are x-intercepts

- VERTEX FORM OF A QUADRATIC FUNCTION: $f(x) = a(x - h)^2 + k$

  Vertex: (h, k) Axis of symmetry: x = h

- To find x-intercepts: Set y = 0, solve for x (or use factored form) :)
- To find y-intercept: Set x = 0, solve for y (or use standard form) :)

- CONCAVITY OF A PARABOLA: *describes whether a parabola opens up or opens down; look at the leading term "a" in the quadratic equation: if it's positive it opens* up *if it's negative it opens* down

Describe each transformation from the parent function $f(x) = x^2$:

1) $f(x) = -3x^2$

V. reflection
V. stretch by 3

2) $g(x) = (x-4)^2 + 1$

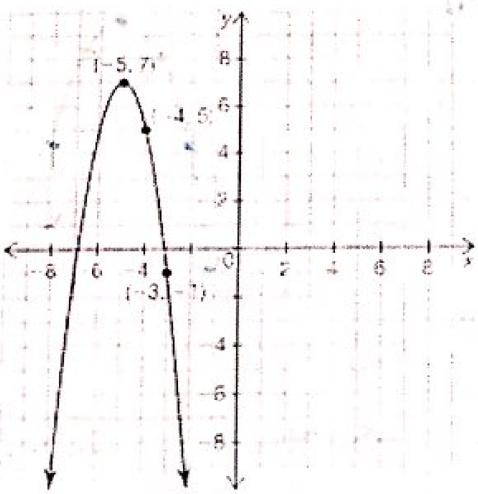
right 4
up 1

3) $h(x) = \frac{1}{3}x^2 - 2$

V. shrink by $\frac{1}{3}$
down 2

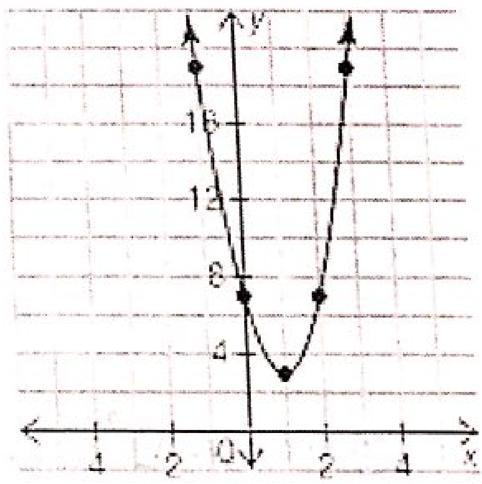
Write the function that represents each graph:

4)



$y = -2(x+5)^2 + 7$

5)



$y = 4(x-1)^2 + 3$

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If a quadratic function is in standard form and you need to graph the function....you need to *complete the square.*

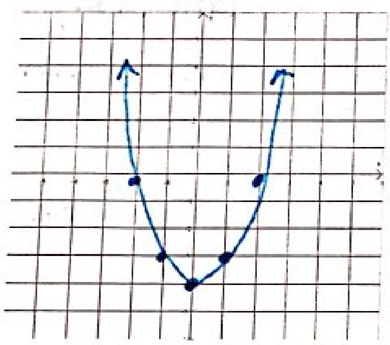
Completing the Square: 1) *Group the "x" terms together...move all other terms to the other side.*
2) *Make a hole after the "x" term and on the other side of the =*
3) *Fill the hole: take ½ the "x" term and square it (add $\left(\frac{b}{2}\right)^2$ to both sides)*
4) *Factor the trinomial (the side with the "x" terms)*
5) Solve for y.

** if "a" is anything other than 1 when you start…. Factor "a" out of the "x"'s and be careful!!

* only need to complete the square if have x^2 and x term

Graph by hand. *Check using your calculator.*

6) $f(x) = x^2 - 4$ ↓4 (* don't need to complete □)



7) $g(x) = x^2 + 6x + 9$

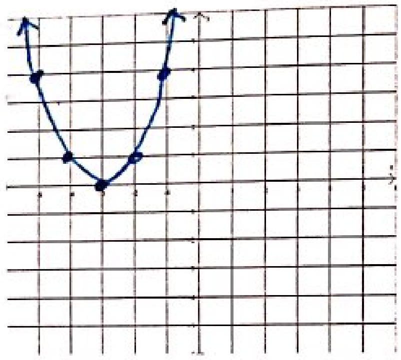
$\left(\frac{6}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$

$y - 9 + \_ = x^2 + 6x + \_$

$y - 9 + \underline{9} = x^2 + 6x + \underline{9}$

$y = (x+3)^2$

← 3



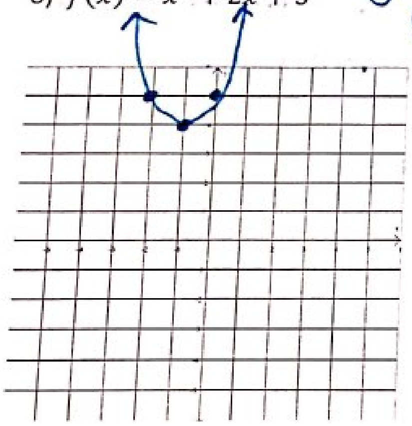
8) $f(x) = x^2 + 2x + 5$

$y - 5 + \underline{1} = x^2 + 2x + \underline{1}$

$y - 4 = (x+1)^2$

$y = (x+1)^2 + 4$

← 1 ↑4



9) $h(x) = -x^2 + 6x - 8$

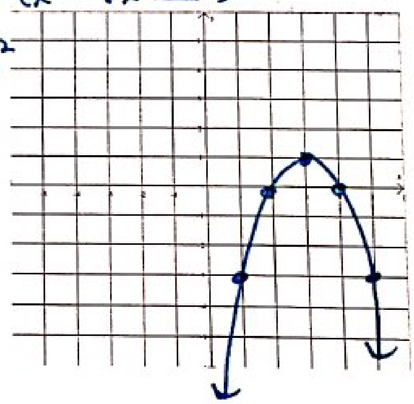
$y + 8 + \underline{-9} = -(x^2 - 6x + \underline{9})$

$y - 1 = -(x-3)^2$

$y = -(x-3)^2 + 1$

→3 ↑1

∩



10) Write the vertex form of the equation of the parabola whose vertex is (1,2) and that passes through (3,-6).

$y = a(x-1)^2 + 2$ $\quad -6 = a(3-1)^2 + 2$ $\quad -6 = 4a + 2$ $\quad -8 = 4a$ $\quad -2 = a$ $\quad \boxed{y = -2(x-1)^2 + 2}$

11) Write two quadratic equations, one that opens up and one that opens down, whose graphs have the given x-intercepts: *(-2,0) & (5,0)*

opens ↑

$y = (x+2)(x-5)$

opens ↓

$y = -(x+2)(x-5)$

12) The total revenue R (in thousands of dollars) earned from manufacturing and selling hand-held video games is given by $R(p) = -25p^2 + 1200p$. (p = price per unit)

a) Find the revenue when the price per unit is \$20 and \$25.

$R(20) = \$14{,}000$ / $R(25) = \$14{,}375$

b) Find the unit price that will yield a max revenue.

\$24 (x-value of maximum – use calc or use $x=\frac{-b}{2a}$)

c) What is the max revenue?

\$14,400 (y-value of maximum – use calc or $f(\frac{-b}{2a})$)

13) A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45 degrees with respect to the ground. The path of the baseball is given by $f(x) = -0.0032x^2 + x + 3$, where $f(x)$ is the height of the baseball (in feet) and x is the horizontal distance from home plate (in feet). *What is the max height reached by the ball?* = what is max y-value?

x = h. distance
y = height

$\boxed{81.125 \text{ feet}}$

Day 2: **Deriving Quadratic Functions**

1) Write the equation of the quadratic function where the x-intercepts are (3,0) & (9,0).

$y = (x-3)(x-9)$

$\boxed{y = x^2 - 12x + 27}$

2) Explain why there are many possible answers to question #1.

* "a" could have any value.

Write the equation for a quadratic function with the following characteristics: Leave in factored form

3) x-intercepts are (-2,0) & (2,0) and (-1,-6) is a point on the parabola.

$y = a(x+2)(x-2)$

$-6 = a(-1+2)(-1-2)$

$-6 = a(1)(-3)$

$-6 = -3a$

$2 = a$

$\boxed{y = 2(x+2)(x-2)}$

4) vertex (-3,4) and point at (-4,1) – Vertex form

$y = a(x+3)^2 + 4$

$1 = a(-4+3)^2 + 4$

$1 = a + 4$

$-3 = a$

$\boxed{y = -3(x+3)^2 + 4}$

5) vertex (3,-2) and an x-intercept at (4,0) – Vertex form

$y = a(x-3)^2 - 2$

$0 = a(4-3)^2 - 2$

$0 = a - 2$

$2 = a$

$\boxed{y = 2(x-3)^2 - 2}$

☻ You can use a graphing calculator to find an equation for a parabola when you are given three points.
Calculator instructions are on page 105 in your Carnegie book. * Store Reg EQ: Y1 makes it easier to graph

Use a calculator to determine the quadratic function for each set of points.

6) (-1,36), (1,12), & (2,6)

$\boxed{y = 2x^2 - 12x + 22}$

7) (0,2), (-1,9), &(3,5)

$\boxed{y = 2x^2 - 5x + 2}$

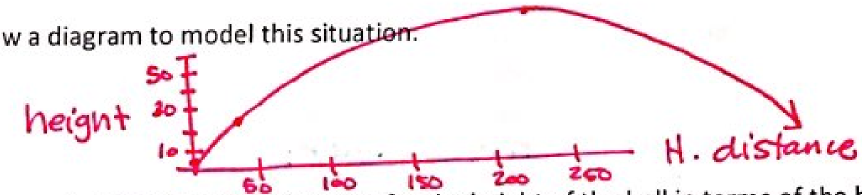
8) (2,3), (3,13) & (4,29)

$\boxed{y = 3x^2 - 5x + 1}$

Van McSlugger needs one more homerun to advance to the next round of the home run derby. On the last pitch, he takes a swing and makes contact. Initially, he hits the ball at 3 feet above the ground. At 32 feet from home plate his ball was 23.7 feet in the air, and at 220 feet from home plate his ball was 70 feet in the air.

(0,3)
(32,23.7)
(220,70)

9) Draw a diagram to model this situation.



10) Use a calculator to write a function for the height of the ball in terms of the horizontal distance. *Round to the nearest thousandth.*

$y = -.002x^2 + .705x + 3$

11) If Van's ball needs to travel a distance of 399 feet in order to get the homerun, did he succeed? *Explain.* graph & find zeros

* Ball hits the ground @ 391.46 feet - No homerun

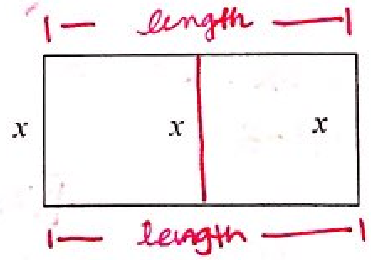
12) What was the max height of Van's baseball?

71.27 feet

Maximum Area Problems:

13) Two adjacent, rectangular corrals are to be made with 600 feet of fencing. Label the sides in terms of x, write an equation for the area of the corral, and find the dimensions for the maximum area.

- Length = $\frac{600-3x}{2}$
- Area = $x\left(\frac{600-3x}{2}\right)$
- graph function -
  y is max area
  x is one of the dimensions of the rectangle (width)



$\boxed{\text{Dimensions: } 100 \times 150}$

max point: (100, 15000)
if x=100 then length = $\frac{600-3(100)}{2} = 150$

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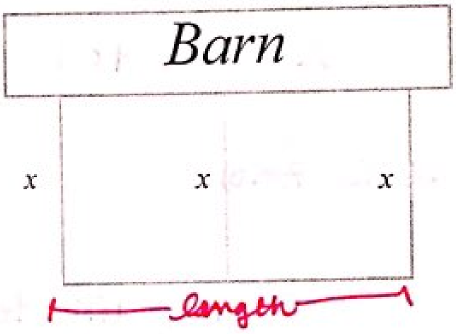
14) Two adjacent rectangular corrals are to be built next to an existing barn with 600 feet of fencing. Label the sides in terms of x, write an equation for the area of the corral, and find the dimensions for the maximum area.

• Length: $\ell = 600 - 3x$

• Area: $A = x(600-3x)$

$= 600x - 3x^2$

• max point: $(100, 30{,}000)$



$\boxed{\text{Dimensions: } 100 \times 300}$

length $= 600 - 3(100) = 300$

Day 3: Complex Number Operations

• The **imaginary number *i*** is a number such that $\boxed{i^2 = -1}$ So what is the value of *i*? $i = \sqrt{-1}$

* Use the value of *i* and i^2 to calculate each power of *i*.

$i = \boxed{i}$	$i^2 = \boxed{-1}$	$i^3 = i \cdot i^2 = i(-1) = \boxed{-i}$	$i^4 = i^2 \cdot i^2 = \boxed{1}$
$i^5 = i^2 \cdot i^2 \cdot i = -1(-1)i = \boxed{i}$	$i^6 = i^2 \cdot i^2 \cdot i^2 = \boxed{-1}$	$i^7 = i^2 \cdot i^2 \cdot i^2 \cdot i = \boxed{-i}$	$i^8 = i^2 \cdot i^2 \cdot i^2 \cdot i^2 = \boxed{1}$
Do you notice a pattern? repeats in blocks of 4 – $i, -1, -i, 1$			

EX) Calculate each power of *i*:

* divide by 4 and use remainder

1) $i^{400} = i^0 = \boxed{1}$

$400 \div 4 = 100 \quad r = 0$

2) $i^{93} = i^1 = \boxed{i}$

$93 \div 4 = 23 \; r \; 1$

3) $i^{206} = i^2 = \boxed{-1}$

$206 \div 4 = 51 \; r \; 2$

4) $i^{-2} = \dfrac{1}{i^2} = \dfrac{1}{-1} = \boxed{-1}$

* don't leave i in denominators.

- **COMPLEX #:** *a number that contains a real part and an imaginary part*
  - Standard Form $a + bi$ a = real part bi = imaginary part
  - *Can every real # be written as a complex #? If so – how?*

    Yes... 2 = 2+0i
- **PURE IMAGINARY #:** *a number in the form bi*

  (a is zero)
- **OPERATIONS WITH COMPLEX #S:**
  - To **add** complex #s: Combine like terms
  - To **subtract** complex #s: Combine like terms – remember to distr. subtraction
  - To **multiply** complex #s: distribute – FOIL

EX) Perform each operation and write in *standard form*: $(a+bi)$

5) $(3-i)-(2+3i)$

$= 3-i-2-3i$

$= \boxed{1-4i}$

6) $\sqrt{-4}+(-4-\sqrt{-4})$

$= 2i+-4-2i$

$= \boxed{-4}$

7) $\sqrt{-4}\cdot\sqrt{-16}$

$= 2i\cdot 4i$

$= 8i^2 = \boxed{-8}$

8) $(\sqrt{-6})^2 = (\sqrt{6}i)^2 = 6i^2$

$= \boxed{-6}$

9) $(2-3i)(4+3i)$

$= 8+6i-12i-9i^2$ (+9)

$= \boxed{17-6i}$

10) $(4+5i)^2 = (4+5i)(4+5i)$

$= 16+20i+20i+25i^2$ (−25)

$= \boxed{-9+40i}$

11) $\frac{1-\sqrt{-44}}{2} = \frac{1}{2} - \frac{\sqrt{44}i}{2}$

$= \frac{1}{2} - \frac{2\sqrt{11}i}{2}$

$= \boxed{\frac{1}{2} - \sqrt{11}i}$

12) $\sqrt{-45}+\sqrt{-25}$

$= \sqrt{45}i + \sqrt{25}i$

$= 3\sqrt{5}i + 5i$

$= \boxed{(3\sqrt{5}+5)i}$

13) $4-3i(5-2i)$

$= 4-15i+6i^2$ (6)

$= \boxed{-2-15i}$

14) $(3+2i)(3-2i)$

$= 9-6i+6i-4i^2$

$= 9+4 = \boxed{13}$

* *Do you notice anything about problem #14?*

Complex #'s are the same except the sign of i ... answer is $\mathbb{R}$

☻ COMPLEX CONJUGATES: *a pair of complex #s in the form of a + bi and a − bi*

- The product of complex conjugates is a real #.

Write the complex conjugate of the complex # and then calculate each product.

15) $(4+3i)(4-3i)$
$= 16 + 12i - 12i - 9i^2$
$= 16 + 9 = \boxed{25}$

16) $(-3+\sqrt{2}i)(-3-\sqrt{2}i)$
$= 9 + 3\sqrt{2}i - 3\sqrt{2}i - 2i^2$
$= 9 + 2 = \boxed{11}$

☻ QUOTIENTS OF COMPLEX #S: *to write complex # quotients in standard form you multiply the numerator and denominator by the conjugate of the* ***denominator***

17) Write in standard form: $\frac{(2+3i)}{(4-2i)} \cdot \frac{(4+2i)}{(4+2i)} = \frac{8+4i+12i+6i^2}{16-4i^2}$

$=0$

$= \frac{2+16i}{20}$

$= \frac{2}{20} + \frac{16}{20}i = \boxed{\frac{1}{10} + \frac{4}{5}i}$

18) Perform the operation and write in standard form: $\frac{5}{3-i} - \frac{4}{3+i}$

$= \frac{5}{(3-i)} \cdot \frac{(3+i)}{(3+i)} - \frac{4}{(3+i)} \cdot \frac{(3-i)}{(3-i)}$

$= \frac{15+5i}{9-i^2} - \frac{12-4i}{9-i^2}$

$= \frac{15+5i}{10} - \frac{(12-4i)}{10}$

$= \frac{15+5i-12+4i}{10} = \frac{3+9i}{10} = \boxed{\frac{3}{10} + \frac{9}{10}i}$

Day 4 : Quadratics & Complex Numbers

To solve a quadratic equation you can:

- Isolate the variable and square root both sides (remember $\pm\sqrt{\ }$) - only if no "x" term
- Factor and use the zero-product property (must =0 first before factoring)
- Use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solve the following quadratic equations using the given method.

1) Square root both sides.

$x^2 + 12 = 0$

$x^2 = -12$

$x = \pm\sqrt{-12}$

$\boxed{x = \pm 2i\sqrt{3}}$

2) Square root both sides.

$4(x-2)^2 - 12 = 0$

$4(x-2)^2 = 12$

$(x-2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

$\boxed{x = 2 \pm \sqrt{3}}$

3) Factoring. (trinomial)

$4x^2 - 12x = -9$

$4x^2 - 12x + 9 = 0$

1,4 2,2 $(2x-3)(2x-3)=0$ 1,9 3,3

$-12x$

$2x-3=0 \quad 2x-3=0$

$\boxed{x = 3/2}$

4) Factoring. (diff of □'s)

$9x^2 - 25 = 0$

$(3x+5)(3x-5) = 0$

$3x+5=0 \quad 3x-5=0$

$\boxed{x = -5/3 \quad x = 5/3}$

5) Quadratic Formula.

$0 = 2x^2 + 8x + 7$

$$x = \frac{-8 \pm \sqrt{64-4(2)(7)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{8}}{4} = \frac{-8 \pm 2\sqrt{2}}{4} = \boxed{\frac{-4 \pm \sqrt{2}}{2}}$$

Which method works the best?

* it depends on the equation.

- The expression $b^2 - 4ac$ is called the discriminant.

If $b^2 - 4ac = 0$, then the quadratic equation has one $\mathbb{R}$ solution (one x-int)

If $b^2 - 4ac > 0$, then the quadratic equation has two $\mathbb{R}$ solutions (two x-int)

If $b^2 - 4ac < 0$, then the quadratic equation has no $\mathbb{R}$ solutions (no x-int)

$\downarrow$ 2 imag. Solutions

Use the discriminant to determine whether the function has real or imaginary roots.

6) $f(x) = -3x^2 + 2x - 1$

$b^2 - 4ac = 4 - 4(-3)(-1)$

$= -8$ / $\boxed{2 \text{ imag}}$

7) $f(x) = -\frac{1}{2}x^2 + x - \frac{1}{2}$

$b^2 - 4ac = 1^2 - 4(-1/2)(-1/2)$

$= 1 - 1 = 0$ / $\boxed{1\ \mathbb{R}}$

8) $g(x) = 2x^2 - 5x - 6$

$b^2 - 4ac = (-5)^2 - 4(2)(-6)$

$= 73$ / $\boxed{2\ \mathbb{R}}$

☻ **THE FUNDAMENTAL THEOREM OF ALGEBRA:** *Any polynomial of degree n must have exactly n complex roots*

$\downarrow$ deg = # of Solutions (roots, zeros)

Find all of the zeros of the function and write the polynomial as a product of linear factors. $(x^1 \pm \#)$
Determine the type of zeros (roots) for each function.

9) $f(x) = x^2 - 12x + 26$

$0 = x^2 - 12x + 26$

$x = \dfrac{12 \pm \sqrt{144 - 4(1)(26)}}{2(1)}$

$= \dfrac{12 \pm \sqrt{40}}{2} = \dfrac{12 \pm 2\sqrt{10}}{2}$

$= \boxed{6 \pm \sqrt{10}}$ zeros / $(x - 6 + \sqrt{10})(x - 6 - \sqrt{10})$

$2\ \mathbb{R}$

10) $g(x) = x^2 - 64$

$0 = x^2 - 64$

$= (x+8)(x-8)$

$\boxed{x = 8, -8}$

$\boxed{(x-8)(x+8)}$

$\boxed{2\ \mathbb{R}}$

11) $h(x) = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

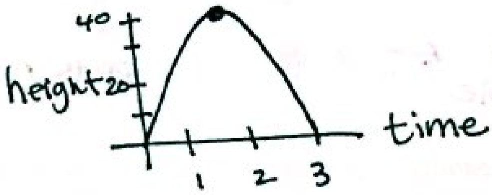
$\boxed{x = 6, -1}$

$\boxed{(x-6)(x+1)}$

$\boxed{2\ \mathbb{R}}$

12) A baseball is thrown upward from ground level with an initial velocity of 48 feet per second, and its height h (in feet) is given by $h(t) = -16t^2 + 48t$, $0 \le t \le 3$ where t is the time (in seconds). You are told that the ball reaches a height of 64 feet. Is this possible? Explain.

max height = 36 feet

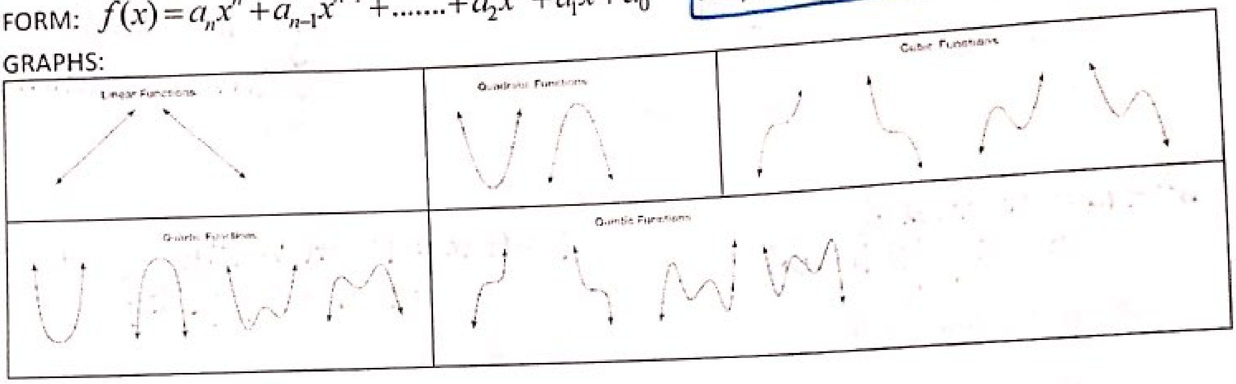


$\boxed{\text{No, not possible}}$

Day 5: **Polynomial Functions of Higher Degrees**

☻ POLYNOMIAL FUNCTIONS: *their graphs are continuous (no holes, breaks, or gaps) and smooth*

- FORM: $f(x)=a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$ $\boxed{n = \text{positive integer}}$
- GRAPHS:



Use your graphing calculator to help you answer the following:

* What shape would $f(x)=x^2; f(x)=x^4; f(x)=x^6, etc.$ make?

* What shape would $f(x)=x^3; f(x)=x^5; f(x)=x^7, etc.$ make?

☻ LEADING COEFFICIENT TEST:
helps to describe a graph's behavior on the left & right hand side of graph (End Behavior)

in front of highest powered term

*highest exponent on variable

	Positive	Negative
Lead coeff	right ↑	right ↓
	Odd	**Even**
Degree	ends opposite	ends same

Use the Leading Coefficient Test to describe the left and right hand side behavior. (end behavior)

1) $f(x) = -x^3 + 4x$
LC: −
Deg: odd
$\boxed{\uparrow \ \downarrow}$

2) $g(x) = 2x^4 - x$
LC: +
Deg: even
$\boxed{\uparrow \uparrow}$

3) $h(x) = x^5$
LC: +
deg: odd
$\boxed{\downarrow \uparrow}$

- For a polynomial of degree *n*, the polynomial (*the function*) has **at most** *n* real zeros (x-intercepts)
- For a polynomial of degree *n*, the graph has **at most** *(n – 1)* relative maximums / minimums (turns)

Also use these facts to help when graphing ☺

REPEATED ZEROS: *For a polynomial function, a factor of* $(x-a)^k$ *has a* **zero** *(or solution or x-intercept or root) at x = a and a* **multiplicity** *of k. (the number of times the factor that created a zero appears)* = multiplicity

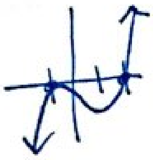
4) Use your calculator graph the functions. State the zeros and state the multiplicity of each zero.

a. $g(x)=(x+1)(x+1)(x-2)$

zeros & multiplicty:

Zeros: -1 m: 2

2 m: 1

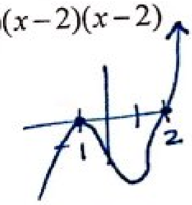


b. $f(x)=(x+1)(x+1)(x-2)(x-2)(x-2)$

zeros & multiplicty:

Zeros: -1 m: 2

2 m: 3



- If the multiplicity of a zero is **odd**, the graph crosses the x-axis @ that point
- If the multiplicity of a zero is **even**, the graph "bounces" off the x-axis @ that point

Find the zeros and determine the multiplicity of each zero. (use factoring ....)

5) $f(x) = x^3 - x^2 - 2x$

$0 = x(x^2-x-2)$

$= x(x-2)(x+1)$

$\boxed{\begin{array}{l} x=0 \quad m: 1 \\ 2 \quad m: 1 \\ -1 \quad m: 1 \end{array}}$

6) $f(x) = -2x^4 + 2x^2$

$0 = -2x^2(x^2-1)$

$= -2x^2(x+1)(x-1)$

$\boxed{\begin{array}{l} \text{Zeros: } 0 \quad m: 2 \\ 1 \quad m: 1 \\ -1 \quad m: 1 \end{array}}$

Find a polynomial function with $\boxed{\text{integer}}$ coefficients that has the following zeros. Remember to simplify.

7) 0, -4, 7

$= x(x+4)(x-7)$

$= x(x^2-3x-28)$

$= \boxed{x^3-3x^2-28x}$

8) 6, -2, ½

$= (x-6)(x+2)(2x-1)$

$= (x^2-4x-12)(2x-1)$

$= 2x^3-x^2-8x^2+4x-24x+12$

$= \boxed{2x^3-9x^2-20x+12}$

9) 2, 3*i*

10) Find a polynomial function with the following: *Zeros: -4 (multiplicity 2); 3 (multiplicity 2)* Degree: 4

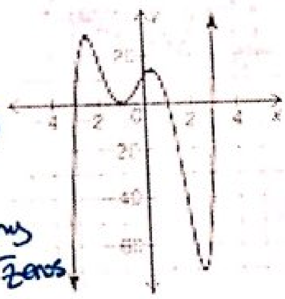
$= (x+4)^2(x-3)^2$

$= (x^2+8x+16)(x^2-6x+9)$

$= x^4-6x^3+9x^2+8x^3-48x^2+72x+16x^2-96x+144$

$= \boxed{x^4+2x^3-23x^2-24x+144}$

11) Use the graph to answer the following:



a) Is the *a-value* of this function positive or negative? (+)

b) Is the degree of the function even or odd? odd

c) Can this function be a cubic? Why or why not? No too many turns/zeros

d) State the domain and range.

D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

e) Determine the number of **relative extrema** and **absolute extrema** in this graph.

(4) (none)

Day 6: More Graphing Polynomial Functions

☻ If f is a polynomial function and a is a real number, then the following statements are equivalent:

- ➡ $x = a$ is a **solution** of the polynomial equation $f(x) = 0$
- ➡ $x = a$ is a **zero or root** of the polynomial equation $f(x) = 0$
- ➡ $(a,0)$ is an **x-intercept** of the graph of $f(x)$
- ➡ $(x - a)$ is a **factor** of $f(x)$

Evaluate the following and discuss your results.

1) Factor: $x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= \boxed{x(x-2)(x+1)}$

2) Solve: $x^3 - x^2 - 2x = 0$

equation

$x(x-2)(x+1) = 0$

$\boxed{x=0 \quad x=2 \quad x=-1}$

3) Find the zeros of $f(x) = x^3 - x^2 - 2x$.

$0 = x(x-2)(x+1)$

$\boxed{x = 0, 2, -1}$

4) Find the x-intercepts of $f(x) = x^3 - x^2 - 2x$.

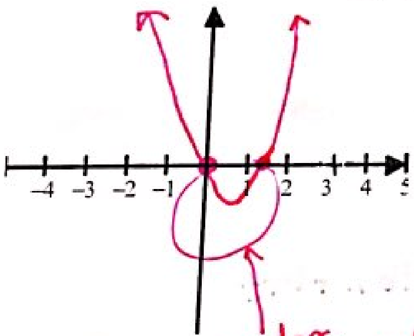
(points)

$0 = x(x-2)(x+1)$

$\boxed{(0,0) \quad (2,0) \quad (-1,0)}$

Graph the following polynomials **by hand**. Use the Leading Coefficient Test, zeros (with multiplicity), and max # of turns to help you.

5) $f(x) = 3x^4 - 4x^3$



*does not need to be exact

EB ↑↑

+ / even

max #: 3

Zeros

$0 = x^3(3x-4)$

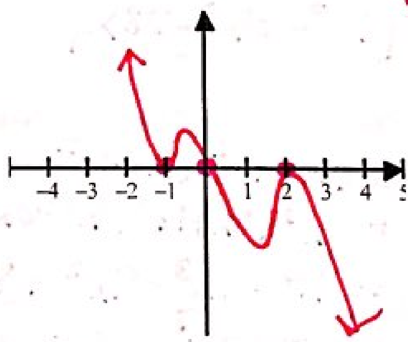
$x = 0$ m: 3

$\frac{4}{3}$ m: 1

6) $g(x) = -2x(x-2)^2(x+1)^2$

deg = 5

$LC = -2(1)^2(1)^2 = -2$



max #: 4 turns

EB: ↑↓

− / odd

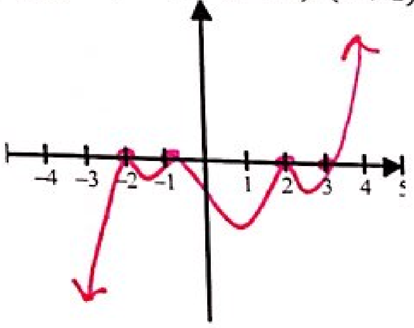
Zeros: $x = 0$ m: 1

2 m: 2

−1 m: 2

$d = 7 \quad LC = \overset{(1)^2(1)^2(1)^2(1)}{1}$

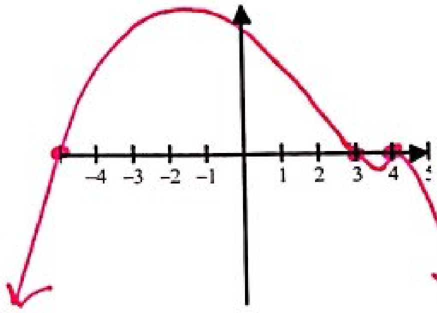
7) $f(x) = (x-2)^2(x+2)^2(x+1)^2(x-3)$



EB ↓↑
max# 6
Zeros
2 m: 2 B
-2 m: 2 B
-1 m: 2 B
3 m: 1 C

$d = 10 \quad LC = \overset{(-1)^2(-1)^3(1)^5}{-1}$

8) $h(x) = (4-x)^2(3-x)^3(x+5)^5$

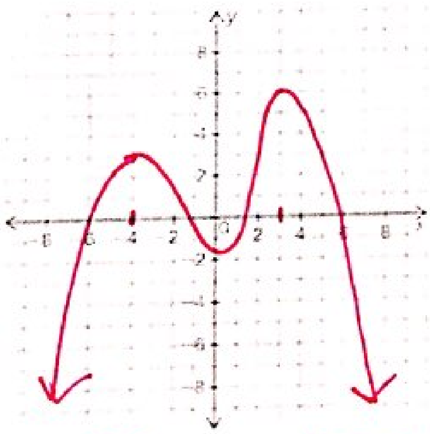


EB
↓↓
max: 9
Zeros
4 m: 2 B
3 m: 3 C
-5 m: 5 C

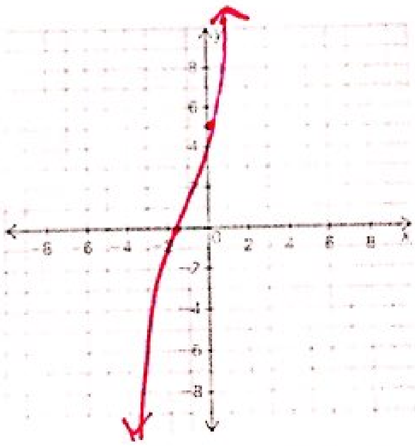
9) Make a number line for the function in #8 to show where *h(x)* is positive and negative.

Use the coordinate plane to sketch a graph with the characteristics given. If the graph is not possible to sketch, explain why.

10) *Degree 4*
Starts in quad III
Ends in quad IV
Relative max at x = -4
Absolute max at x = 3

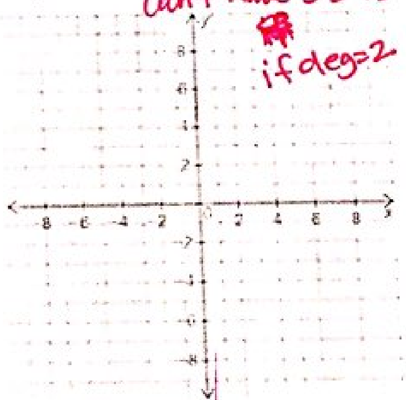


11) *Always increasing*
y-intercept at 5
x-intercept at -1.8



12) *Degree 2*
negative "a" value
x-int at -2, 2, & 5

Not possible
Can't have 3 zeros
if deg=2



Write a cubic function with integer coefficients that has the following characteristics:

13) zeros: x = -3, 0, 1

$= (x+3)(x)(x-1)$
$= x(x^2+2x-3)$
$= \boxed{x^3+2x^2-3x}$

14) zeros: x = 0, -3i, also $-3i$

$= x(x+3i)(x-3i)$
$= x(x^2-9i^2)$
$= x(x^2+9) = \boxed{x^3+9x}$

15) zeros: x = 5 (mult 2), x = -1

$= (x-5)^2(x+1)$
$= (x^2-10x+25)(x+1)$
$= x^3+x^2-10x^2-10x+25x+25$
$= \boxed{x^3-9x^2+15x+25}$

16) zeros: x = -1, -5, 2 & y-int (0,-20)

$y = a(x+1)(x+5)(x-2)$
$-20 = a(0+1)(0+5)(0-2)$
$-20 = -10a$
$2 = a$

$y = 2(x+1)(x+5)(x-2)$
$= 2(x^2+6x+5)(x-2)$
$= 2(x^3-2x^2+6x^2-12x+5x-10)$
$= 2(x^3+4x^2-7x-10)$
$= \boxed{2x^3+8x^2-14x-20}$