Unit 5 Notes
Rational Expressions and Functions

Day 1
Simplify Rational Expressions

Reducing a fraction means to find an equivalent fraction with no common factors in numerator and denominator.

Which of the following equivalent fractions is reduced? $\frac{21}{28}, \frac{15}{20}, \frac{3}{4}, \frac{x^2}{2x}$

1. Reduce the following fractions: $\frac{15 \div 5}{40 \div 5} = \frac{3}{8}$ $\quad \frac{24y \div 6}{42y^2 \div 6} = \frac{4}{7y}$ $\quad \frac{3+6}{3-1} = \frac{9}{2}$

Simplifying rational expressions is the same process as reducing fractions: divide out common **factors.**

Examples. Simplify:

2. $\frac{\overset{2}{\cancel{12}}x^3}{\underset{5}{\cancel{30}}x} = \frac{2x^2}{5}$

3. $\frac{\overset{1}{\cancel{16}}x^2y^3}{\underset{2}{\cancel{32}}x^3y^2} = \frac{y}{2x}$

4. $\frac{\cancel{x^2}(\cancel{x+2})(\cancel{x-4})(x-3)}{\cancel{x}(x-4)^{\cancel{2}}(\cancel{x+2})}$

$= \frac{x(x-3)}{x-4}$

But BE CAREFUL. Can you simplify $\frac{15+x^3}{3-x}$? No!!! +'s and -'s.

Rational expressions usually have restrictions on what x (or any other variable) can be. A variable cannot be a number that will create a 0 denominator. What are the restrictions on the rational expression 4 above? (Look at the expression BEFORE it is simplified. Each factor in the denominator might create a restriction.)

ALWAYS factor as much as possible BEFORE dividing out common factors.

Examples. For each expression, factor, list the restrictions of x and simplify:

5. $\frac{x^2-8x+12}{x^2+4x-12}$

$= \frac{(x-6)(\cancel{x-2})}{(x+6)(\cancel{x-2})}$ $\quad x \neq -6, 2$

$= \boxed{\frac{x-6}{x+6}}$

6. $\frac{x^3+3x^2-x-3}{x^2+2x-3}$

Top:
$= (x^3+3x^2)+(-x-3)$
$= x^2(x+3)-1(x+3)$
$= (x+3)(x^2-1)$
$= (x+3)(x+1)(x-1)$

$(x+3)(x-1)$

$\frac{(\cancel{x+3})(x+1)(\cancel{x-1})}{(\cancel{x+3})(\cancel{x-1})}$ $\quad x \neq -3, 1$

$= \boxed{x+1}$

Multiply Rational Expressions

Multiply fractions by multiplying the numerators and the denominators.

7. Multiply: $\frac{4 \to 5}{15 \to 8}$ $= \frac{20}{120} = \frac{1}{6}$

If you divide out common factors BEFORE multiplying, you will not have to reduce (simplify) afterward.

8. Divide out common factors, then multiply: $\frac{\cancel{4}}{\cancel{15}\,3} \cdot \frac{\cancel{5}}{\cancel{8}\,2} = \frac{1}{6}$

To multiply rational expressions:

Steps
- Factor each fraction as much as possible.
- List restriction(s) on the variable(s) for the denominator of each fraction.
- Divide out common factors.
- Multiply; leave answers in factored form.

Examples. Multiply and simplify. List all restrictions on the variable(s).

9. $\frac{3xy^3}{8z^3} \cdot \frac{4x}{21x^3}$ $X \neq 0 \quad Y \neq 0 \quad Z \neq 0$

$= \boxed{\frac{Y}{14xz^2}}$

10. $\frac{5x}{6x-18} \cdot \frac{x-3}{10x^2}$ $X \neq 3 \quad X \neq 0$

$6(x-3)$

$= \frac{1}{12x}$

11. $\frac{2x^3}{9} \cdot \frac{x^2-4x+3}{x-7} \cdot \frac{x^2-4x-21}{6x^2-18x}$ $X \neq 7, 0, 3$

$= \frac{2x^{3}}{9} \cdot \frac{\cancel{(x-3)}(x-1)}{\cancel{x-7}} \cdot \frac{\cancel{(x-7)}(x+3)}{\cancel{6}x\cancel{(x-3)}}$

$= \boxed{\frac{x^2(x-1)(x+3)}{27}}$

Day 2

Divide Rational Expressions

To divide fractions, multiply by the reciprocal of the divisor. Again, if you divide out common factors before multiplying, you won't have to reduce.

1. Divide:

a. $\dfrac{\frac{2}{3}}{\frac{6}{11}}$ $\quad \dfrac{\cancel{2}}{3} \cdot \dfrac{11}{\cancel{6}\,3} = \boxed{\dfrac{11}{9}}$

b. $\dfrac{\frac{5}{6}}{15}$ $\quad \dfrac{\cancel{5}}{6} \cdot \dfrac{1}{\cancel{15}\,3} = \boxed{\dfrac{1}{18}}$

To divide rational expressions:

- Write as a multiplication.
- Factor all fractions as much as possible.
- ✳ List all restrictions in each denominator, including the denominator of the divisor in the original.
- Divide out common factors.
- Multiply; leave answer in factored form.

Examples. Divide and simplify the rational expressions. List all restrictions on the variable(s):

2. $\dfrac{2x^4}{3y^2} \div \dfrac{8x^3}{21y}$

$\dfrac{\cancel{2}x^{\cancel{4}}}{\cancel{3}y^{\cancel{2}}} \cdot \dfrac{\cancel{21}^{7}\cancel{y}}{\cancel{8}_{4}\cancel{x^3}}$ $\quad x \neq 0 \quad y \neq 0$

$= \boxed{\dfrac{7x}{4y}}$

3. $\dfrac{x^2+7x+6}{x^2-16} \div \dfrac{x^2+3x-18}{x^2-5x+4}$

$\dfrac{(x+6)(x+1)}{(x+4)(x-4)} \cdot \dfrac{x^2-5x+4}{x^2+3x-18}$

$= \dfrac{\cancel{(x+6)}(x+1)}{(x+4)\cancel{(x-4)}} \cdot \dfrac{\cancel{(x-4)}(x-1)}{\cancel{(x+6)}(x-3)}$ $\quad x \neq -4, 4, -6, 3, 1$

$= \boxed{\dfrac{(x+1)(x-1)}{(x+4)(x-3)}}$

FIX 4. $\dfrac{5xy}{\cancel{2-x}\ x-2} \div \dfrac{10y}{\cancel{4-x^2}\ x^2-4} \cdot \dfrac{3}{\cancel{2+x}\ x+2}$ $\quad x \neq 2, -2 \quad y \neq 0$

$= \dfrac{5xy}{x-2} \cdot \dfrac{x^2-4}{10y} \cdot \dfrac{3}{x+2}$

$= \dfrac{\cancel{5}x\cancel{y}}{\cancel{x-2}} \cdot \dfrac{\cancel{(x-2)}\cancel{(x+2)}}{\cancel{10}_{2}\cancel{y}} \cdot \dfrac{3}{\cancel{x+2}}$

$\boxed{\dfrac{3x}{2}}$

FIX 5. $\dfrac{x^2-4x+3}{-2x^3} \div \dfrac{\cancel{9-x^2}\ x^2-9}{16x^2}$

$= \dfrac{x^2-4x+3}{-2x^3} \cdot \dfrac{16x^2}{x^2-9}$

$= \dfrac{\cancel{(x-3)}(x-1)}{\cancel{-2}x^{\cancel{3}}} \cdot \dfrac{\cancel{16}^{-8}\cancel{x^2}}{(x+3)\cancel{(x-3)}}$ $\quad x \neq 0, 3, -3$

$= \boxed{\dfrac{-8(x-1)}{x(x+3)}}$

Assignment 5.2

Day 3

Add Rational Expressions

need a common denominator

1. Add $\frac{2}{3}$ to $\frac{5}{3}$ $\frac{2}{3} + \frac{5}{3} = \boxed{\frac{7}{3}}$ Now add $\frac{2\cdot 2}{3\cdot 2}$ to $\frac{5}{6}$ $\frac{4}{6} + \frac{5}{6} = \frac{9\div 3}{6\div 3} = \boxed{\frac{3}{2}}$

need a common denominator

2. In the same way, add $\frac{2}{3x}$ to $\frac{5}{3x}$ Now add $\frac{2\cdot x}{3x\cdot x}$ to $\frac{5}{3x^2}$

$\frac{2}{3x} + \frac{5}{3x} = \boxed{\frac{7}{3x}}$ $\frac{2x}{3x^2} + \frac{5}{3x^2} = \boxed{\frac{2x+5}{3x^2}}$

Denominators must be the same to add fractions, so it's critical to identify the common denominator. To save work, find the smallest denominator (LCD or least common denominator) possible.

Important fact: you change denominators by MULTIPLYING, never adding or subtracting. That's because we're trying to find equivalent fractions.

3. Is $\frac{5}{6}$ an equivalent fraction to $\frac{7}{8}$? How about to $\frac{10}{12}$?

$\frac{5+2}{6+2} = \frac{7}{8}$ $\frac{5\cdot 2}{6\cdot 2} = \frac{10}{12}$

No because we added Yes because we multiplied

Example. Identify the LCD for the following sets of fractions. You may have to factor to find the LCD:

4. $\frac{3}{x}$ and $\frac{2}{x+1}$ LCD: $x(x+1)$

* Cannot add

$x(x+1)$

just multiply the two together

5. $\frac{x}{x-2}$ and $\frac{x-4}{x+3}$ LCD: $(x-2)(x+3)$

6. $\frac{3}{x\cdot x}$ and $\frac{6}{x^2}$ LCD: x^2

if we multiply this by x it will be x^2.

7. $\frac{2}{x-7}$ and $\frac{3}{x^2-9x+14}$ always factor first! LCD: $(x-7)(x-2)$

$(x-7)(x-2)$

They both have $(x-7)$ so if I multiply by $(x-2)$ they will match.

To add rational expressions:

- Identify the LCD
- Multiply numerator and denominator of each fraction by any factor(s) it's missing to obtain a common denominator.
- Write as a single numerator over the common denominator.
- Distribute and combine like terms to simplify the numerator. Factor the numerator if possible and simplify (to reduce).
- List any restrictions for the variable.

Examples. Add the following rational expressions. List any restrictions on the variable:

8. $\frac{6 \cdot x}{x \cdot x}+\frac{8}{x^2}$ LCD: x^2

$\frac{6x}{x^2}+\frac{8}{x^2}=\frac{6x+8}{x^2}$ $\boxed{x \neq 0}$

$=\boxed{\frac{2(3x+4)}{x^2}}$

9. $\frac{4}{x^2-1}+\frac{8}{x-1}$ LCD: $(x+1)(x-1)$

$\frac{4}{(x+1)(x-1)}+\frac{8\,(x+1)}{x-1\,(x+1)}$

$=\frac{4}{(x+1)(x-1)}+\frac{8x+8}{(x+1)(x-1)}$ $\boxed{x \neq 1,-1}$

$=\frac{8x+12}{(x+1)(x-1)}=\boxed{\frac{4(2x+3)}{(x+1)(x-1)}}$

10. $\frac{x}{x+4}+\frac{3}{x-4}$ LCD: $(x+4)(x-4)$

$\frac{x\,(x-4)}{(x+4)(x-4)}+\frac{3\,(x+4)}{(x-4)(x+4)}$

$=\frac{x^2-4x+3x+12}{(x+4)(x-4)}$ $\boxed{x \neq -4,4}$

$=\boxed{\frac{x^2-x+12}{(x+4)(x-4)}}$

11. $\frac{x+1}{x^2-4x+3}+\frac{1(x-3)}{x-1(x-3)}+\frac{6(x-1)}{x-3(x-1)}$ LCD: $(x-1)(x-3)$

$(x-1)(x-3)$

$\frac{x+1+x-3+6x-6}{(x-1)(x-3)}$

$=\frac{8x-8}{(x-1)(x-3)}$ $\boxed{x \neq 1,3}$

$=\frac{8\cancel{(x-1)}}{\cancel{(x-1)}(x-3)}$

$=\boxed{\frac{8}{x-3}}$

Assignment 5.3

++

Day 4

Subtract Rational Expressions

Adding rational expressions is the same as subtracting EXCEPT the numerator is negative and that negative needs to be distributed.

Examples. Subtract (or add) the following rational expressions. List any restrictions for the variable:

1. $\frac{6(x+1)}{x-5(x+1)}-\frac{x\,(x-5)}{x+1(x-5)}$ LCD: $(x-5)(x+1)$

$x \neq 5,-1$

$\frac{6x+6-(x^2-5x)}{(x-5)(x+1)}$

$=\frac{6x+6-x^2+5x}{(x-5)(x+1)}$

$=\boxed{\frac{-x^2+11x+6}{(x-5)(x+1)}}$

2. $\frac{2x+1\,(x+1)}{x^2}-\frac{2\,x(x+1)}{x}+\frac{x\,(x^2)}{x+1}$ LCD: $x^2(x+1)$

$x \neq 0,-1$

$\frac{2x^2+2x+x+1-(2x^2+2x)+x^3}{x^2(x+1)}$

$=\frac{\cancel{2x^2}+\cancel{2x}+x+1-\cancel{2x^2}-\cancel{2x}+x^3}{x^2(x+1)}$

$=\boxed{\frac{x^3+x+1}{x^2(x+1)}}$

Simplify Complex Fractions

To simplify complex fractions:

- Add/subtract fractions as necessary until the top and bottom are both a single fraction.
- Change to a multiplication problem, factor if possible and simplify.

Examples. Simplify the following complex fractions:

3. $\dfrac{3+\dfrac{4}{x}}{\dfrac{3x-4}{x}}$

LCD: x

$\dfrac{3x}{x}+\dfrac{4}{x}=\dfrac{3x+4}{x}$ Single Fraction

already a Single Fraction

rewrite: $\dfrac{3x+4}{\cancel{x}}\cdot\dfrac{\cancel{x}}{3x-4}$ $x\neq 0,\frac{4}{3}$

$=\boxed{\dfrac{3x+4}{3x-4}}$

4. $\dfrac{\dfrac{1}{x}+x}{x-\dfrac{1}{x}}$

LCD: x

$\dfrac{1}{x}+\dfrac{x^2}{x}=\dfrac{1+x^2}{x}$ Single Fraction

LCD: x

$\dfrac{x^2}{x}-\dfrac{1}{x}=\dfrac{x^2-1}{x}$ Single Fraction

rewrite: $\dfrac{1+x^2}{\cancel{x}}\cdot\dfrac{\cancel{x}}{x^2-1}$ $x\neq 0,1,-1$

$=\boxed{\dfrac{1+x^2}{x^2-1}}$

5. $\dfrac{\dfrac{2}{x-1}-\dfrac{1}{x}}{\dfrac{1}{x}+\dfrac{2}{x-1}}$

LCD: x(x−1)

$\dfrac{2(x)}{(x-1)x}-\dfrac{1(x-1)}{x(x-1)}$

$\dfrac{2x-x+1}{x(x-1)}=\dfrac{x+1}{x(x-1)}$ Single Fraction

LCD: x(x−1)

$\dfrac{x-1}{x(x-1)}+\dfrac{2x}{(x-1)x}$

$=\dfrac{x-1+2x}{x(x-1)}$

$=\dfrac{3x-1}{x(x-1)}$ Single Fraction

rewrite: $\dfrac{x+1}{\cancel{x(x-1)}}\cdot\dfrac{\cancel{x(x-1)}}{3x-1}$ $x\neq 0,1,\frac{1}{3}$

$=\boxed{\dfrac{x+1}{3x-1}}$

Operations with functions

Examples. Given $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{2}{x}$, find and simplify the following:

6. $f(x)+g(x)$

$\dfrac{x(x)}{x+1(x)}+\dfrac{2(x+1)}{x(x+1)}$ $x\neq 0,-1$

$=\boxed{\dfrac{x^2+2x+2}{x(x+1)}}$

7. $g(x)-f(x)$

$\dfrac{2(x+1)}{x(x+1)}-\dfrac{x\cdot x}{(x+1)x}$

$=\dfrac{2x+2-x^2}{x(x+1)}$ $x\neq 0,-1$

$=\boxed{\dfrac{-x^2+2x+2}{x(x+1)}}$

8. $\dfrac{f(x)}{g(x)}$ $\dfrac{\dfrac{x}{x+1}}{\dfrac{2}{x}}$

rewrite: $\dfrac{x}{x+1}\cdot\dfrac{x}{2}$

$=\boxed{\dfrac{x^2}{2(x+1)}}$ $x\neq -1,0$

Assignment 5.4

Day 5

Solve Rational Equations

Rationa[...] s that are restrict[...] al equation.

To solv[...]

To Solve rational equations:

- If single fractions → cross multiply
- If not... multiply both sides by LCD (this gets rid of fractions)

Then Solve remaining equation

* Check solutions

Examples. Solve the following rational equations:

1. $2x\left[\frac{5x}{2} + \frac{5}{2x} = 5x\right]2x$ $\quad x \neq 0$ $\quad$ LCD: $2x$

$\cancel{2}x\left(\frac{5x}{\cancel{2}}\right) + \cancel{2x}\left(\frac{5}{\cancel{2x}}\right) = 5x(2x)$

$5x^2 + 5 = 10x^2$

$-5x^2$

$\frac{5}{5} = \frac{5x^2}{5}$

$\pm\sqrt{1} = \pm\sqrt{x^2}$

$x = \pm 1$

$\boxed{x = 1, -1}$ Check against restrictions ✓

2. $\frac{x+5}{x+2} = \frac{x+1}{x-5}$ $\quad x \neq -2, 5$

Cross multiply

$(x+5)(x-5) = (x+2)(x+1)$

$x^2 + 5x - 5x - 25 = x^2 + x + 2x + 2$

$\cancel{x^2} - 25 = \cancel{x^2} + 3x + 2$

$-x^2 \quad -x^2$

$-25 = 3x + \cancel{2}$

$-2 \quad -2$

$\frac{-27}{3} = \frac{3x}{3}$

$\boxed{x = -9}$ ✓

3. $x(x-2)\left(\frac{2}{x} = \frac{3}{x-2} - 1\right)x(x-2)$ $\quad x \neq 0, 2$ $\quad$ LCD: $x(x-2)$

$\cancel{x}(x-2)\left(\frac{2}{\cancel{x}}\right) = x\cancel{(x-2)}\left(\frac{3}{\cancel{x-2}}\right) - x(x-2)(1)$

$2x - 4 = 3x - (x^2 - 2x)$

$2x - 4 = 3x - x^2 + 2x$

$2x - 4 = 5x - x^2$

$-5x \quad -5x$

$-3x - 4 = -\cancel{x^2}$

$+x^2 \quad +\cancel{x^2}$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x = 4, -1}$ ✓

4. $\frac{2x+4}{x^2-2x-8} = \frac{x+1}{x-4}$ $\quad x \neq 4, -2$

Cross multiply

$(x-4)(x+2)$

$\frac{(2x+4)\cancel{(x-4)}}{\cancel{x-4}} = \frac{\cancel{(x-4)}(x+2)(x+1)}{\cancel{x-4}}$

$2x + 4 = (x+2)(x+1)$

$\cancel{2x} + \cancel{4} = x^2 + 3x + 2$

$-2x \quad -4 \quad\quad -2x \quad -4$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = \cancel{-2}, \boxed{1}$

extraneous

LCD: $(x-2)(x-2)$

LCD: $x(x-2)$

$x \neq 2, -2$

$x \neq 0, 2$

5. $(x-2)(x+2)\left(\frac{1}{x-2} = \frac{3}{x+2} - \frac{6}{x^2-4}\right)(x-2)(x+2)$

$x^2-4 \to (x-2)(x+2)$

$(x-2)(x+2)\left(\frac{1}{x-2}\right) = (x-2)(x+2)\left(\frac{3}{x+2}\right) - (x-2)(x+2)\left(\frac{6}{(x-2)(x+2)}\right)$

$x+2 = 3x-6-6$

$x+2 = 3x-12$

$-x$

$2 = 2x - 12$

$+12 \quad +12$

$\frac{14}{2} = \frac{2x}{2}$

$\boxed{x=7}$ ✓

6. $x(x-2)\left(\frac{10}{x^2-2x} + \frac{1}{x} = \frac{3}{x-2}\right)x(x-2)$

$x^2-2x \to x(x-2)$

$x(x-2)\left(\frac{10}{x(x-2)}\right) + x(x-2)\left(\frac{1}{x}\right) = x(x-2)\left(\frac{3}{x-2}\right)$

$10 + x - 2 = 3x$

$-x \quad -x$

$8 = 2x$

$\boxed{x=4}$ ✓

Assignment 5.5

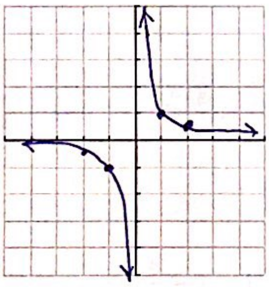
++

Day 6

Rational Functions and Asymptotes

A rational function is a function defined as a ratio of polynomials (there must be an x in the denominator).

1. Graph $y = \frac{1}{x}$



Domain: $x \neq 0, \mathbb{R}$

Range: $y \neq 0, \mathbb{R}$

Vertical Asymptote:

$x = 0$

y-axis

Horizontal Asymptote:

$y = 0$

x-axis

Domain

The domain of a rational function is all real numbers **except** those that will make the denominator zero (restrictions).

2. Find the domain of each rational function:

a. $f(x) = \frac{x+3}{x^2-9}$

$(x+3)(x-3)$

D: $\mathbb{R}$, $x \neq 3, -3$

b. $g(x) = \frac{x-5}{x^2+3x-4}$

$(x+4)(x-1)$

D: $\mathbb{R}$, $x \neq -4, 1$

Vertical Asymptotes

- Simplify the rational expression. x values in the **simplified** function where the denominator is zero are vertical asymptotes.
- A graph can never cross a vertical asymptote! (Why not?) y is undefined there

Horizontal Asymptotes

- Horizontal asymptotes describe the end behaviors of the graph. The graph can cross a horizontal asymptote but not at the ends.
- To find the horizontal asymptote, look at the largest power of x in both numerator and denominator of the expanded (not simplified) fraction.

Numerator < Denominator	$y = 0$
Numerator = Denominator	**y = ratio of leading coefficients**
Numerator > Denominator	**no horizontal asymptote**

Holes

- A gap (missing point) in the graph. Plot with an open circle.
- Find the x-value of holes by setting a crossed-out factor = 0.
- The y-value of the hole must be found by replacing x in the simplified fraction.
- THERE WILL NEVER BE A HOLE AND A VERTICAL ASYMPTOTE AT THE SAME X-VALUE.

***y*-intercept**

Find the y-intercept of a function by replacing x with 0 and evaluating.

***x*-intercept(s)**

Find any x-intercept(s) by setting the function equal to zero and solving.

Examples.

3. Find the vertical asymptotes:

a. $f(x) = \dfrac{5x}{x+7}$ $\boxed{X = -7}$

b. $g(x) = \dfrac{3x-6}{x^2-4} = \dfrac{3\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$ V.A. $\boxed{X = -2}$

c. $h(x) = \dfrac{2x+1}{x^2-5x+6} = \dfrac{2x+1}{(x-3)(x-2)}$

VA: $\boxed{\begin{array}{l} x = 3 \\ x = 2 \end{array}}$

d. $y = \dfrac{x^2-1}{x^2+1} \rightarrow x^2+1=0$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i \Rightarrow$ (imaginary)

$\boxed{\text{no vertical asymptote}}$

4. Determine the horizontal asymptotes:

a. $f(x) = \dfrac{5x}{x+2}$ degree = 1, degree = 1

HA: $\boxed{y = 5}$

b. $h(x) = \dfrac{2x+1}{x^2-5x+6}$ d = 1, d = 2

HA: $\boxed{y = 0}$

c. $y = \dfrac{x^2-1}{x^2+1}$ d = 2, d = 2

HA: $\boxed{y = 1}$

5. Find the hole in the graph of $f(x)=\dfrac{x^2-1}{x^2+3x+2}$ $(-1, -2)$

$= \dfrac{(x-1)\cancel{(x+1)}}{(x+2)\cancel{(x+1)}} = \dfrac{x-1}{x+2}$ $\dfrac{-1-1}{-1+2} = \dfrac{-2}{1} = -2$

$x+1=0$ $x=-1$

6. Find the domain of $f(x)=\dfrac{x^2-4}{3x^2+6x}$. Then determine the vertical asymptote(s), hole(s) in the graph, and the horizontal asymptote for $f(x)$.

$\dfrac{\cancel{(x+2)}(x-2)}{3x\cancel{(x+2)}}$

Domain: $\mathbb{R}$ $x \neq 0, -2$ Simplified function: $\dfrac{x-2}{3x}$

HA: $y=\frac{1}{3}$ VA: $x=0$ Hole: $(-2, 2/3)$

$\dfrac{-2-2}{3(-2)} = \dfrac{-4}{-6} = \dfrac{2}{3}$

7. Find the y-intercept and the x-intercepts of each function.

a. $f(x)=\dfrac{5x}{x+2}$

$\boxed{\text{x-int: } (0,0)}$

$0=\dfrac{5x}{x+2}$

$0=5x$

$x=0$

$\boxed{\text{y-int: } (0,0)}$

$y=\dfrac{5(0)}{0+2}$

$=\dfrac{0}{2}=0$

b. $h(x)=\dfrac{2x+1}{x^2-5x+6}$

$\boxed{\text{x-int: } (-1/2, 0)}$

$0=2x+1$

$-1 \qquad -1$

$-1=2x$

$x=-1/2$

$\boxed{\text{y-int: } (0, 1/6)}$

$y=\dfrac{2(0)+1}{0-0+6} = \dfrac{1}{6}$

c. $y=\dfrac{x^2-1}{x^2+1}$

$\boxed{\text{x-int: } (1,0)(-1,0)}$

$0=x^2-1$

$\pm\sqrt{1}=\sqrt{x^2}$

$x=1,-1$

$\boxed{\text{y-int: } (0,-1)}$

$y=\dfrac{0-1}{0+1}=-1$

8. The cost C (in millions of dollars) to companies for removing $p\%$ of the pollutants discharged into a river is given by the equatio[n] $C=\dfrac{255p}{100-p}$, $0 \le p < 100$

a. The current law requir[es companies to remove 45% of the pollutants.] What is the current cost of removal?

b. A proposed law would [...] [pollutan]ts. What would the cost be under the proposed law?

c. How much additional [...] [la]w passes?

d. One legislator propos[...]. If this is possible, how much would it cost to do so? If no[...]

↖ SKip #8

Do #22, 25, 28 off the assignment as a class

Assignment 5.6

Day 7

Graphs of Rational Functions

To graph rational functions:

- Determine the domain.
- Find horizontal asymptote.
- Factor and simplify to identify vertical asymptote(s) and hole (if there is one).
- Find any intercepts.
- Draw asymptotes as dashed lines and plot intercept(s) and holes where they exist.
- Plot additional points as necessary.
- Draw smooth curves, approaching asymptotes.
- Check to make sure that your graph passes the vertical line test. It needs to be a function.

Examples. Graph the following functions by hand, considering the following aspects:

1. $y = \dfrac{2x}{1-x}$

Domain: $\mathbb{R}$, $x \neq 1$

HA: $y = -2$

VA: $x = 1$

Hole(s): none

y-int: $(0,0)$

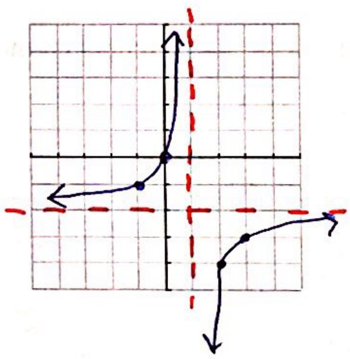
x-int: $(0,0)$

Additional point(s);

Plug in $-1 \rightarrow \dfrac{2(-1)}{1--1} = \dfrac{-2}{2} = -1$ $\boxed{(-1,-1)}$

Plug in $2 \rightarrow \dfrac{2(2)}{1-2} = \dfrac{4}{-1} = -4$ $\boxed{(2,-4)}$

Plug in $3 \rightarrow \dfrac{2(3)}{1-3} = \dfrac{6}{-2} = -3$ $\boxed{(3,-3)}$



2. $f(x) = \dfrac{x^2-1}{x^2-4} = \dfrac{(x+1)(x-1)}{(x+2)(x-2)}$

Domain: $\mathbb{R}$ $x \neq -2, 2$

HA: $y = 1$

VA: $x = 2$ $x = -2$

Hole(s): none

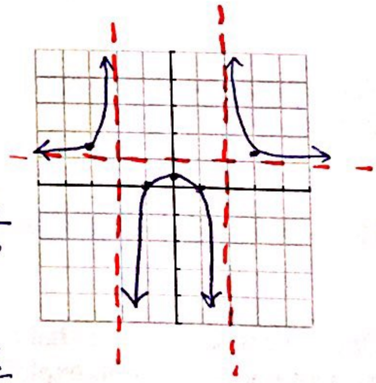
y-int: $(0, 1/4)$ $\dfrac{0-1}{0-4} = \dfrac{1}{4}$

x-int: $(-1, 0)$ $(1, 0)$ $0 = \dfrac{(x+1)(x-1)}{(x+2)(x-2)}$

Additional point(s):

Plug in $-3 \rightarrow \dfrac{(-3)^2-1}{(-3)^2-4} = \dfrac{8}{5}$ $\boxed{(-3, 8/5)}$

Plug in $3 \rightarrow \dfrac{3^2-1}{3^2-4} = \dfrac{8}{5}$ $\boxed{(3, 8/5)}$



3. $f(x)=\frac{x}{(x-2)^2}$

Domain: $\mathbb{R}$, $x \neq 2$

HA: $y=0$

VA: $x=2$

Hole(s): none

y-int: $(0,0)$

x-int: $(0,0)$

Additional point(s):

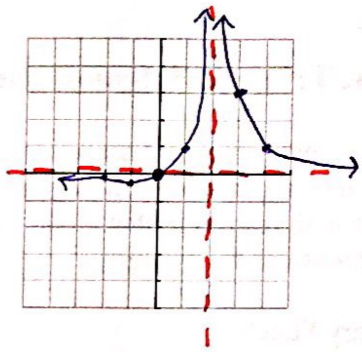
$(-1, -1/9)$

$(1,1)$

$(-2, -1/8)$

$(3,3)$

$(4,1)$



4. $g(x)=\frac{x^2-9}{x^2-2x-3} = \frac{(x+3)(x-3)}{(x-3)(x+1)}$

Domain: $\mathbb{R}$, $x \neq 3, -1$

Simplified function: $g(x)= \frac{x+3}{x+1}$

HA.: $y=1$

VA: $x=-1$

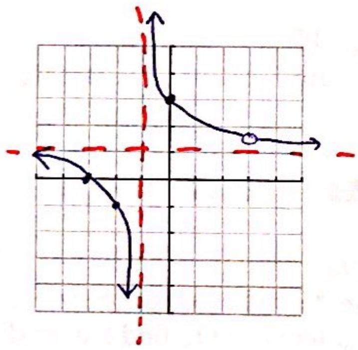
Hole(s): $(3, 3/2)$ $\frac{3+3}{3+1} = \frac{6}{4} = \frac{3}{2}$

y-int: $(0,3)$

x-int: $(-3,0)$

Additional point(s):

$(-2,-1)$



Check with your calculator. Again, your calculator can verify the general shape of your graph, but it is inadequate in showing holes and sometimes graphical behavior close to asymptotes.

Assignment 5.7

++

Day 8

Unit 5 Review

Assignment 5.8

++

Day 8

Unit 5 Test ***All late/absent assignments due for Unit 5***

++