Key

Unit 9 Notes / Secondary 3 Honors

Day 1: **Basic Trigonometric Identities**

What's an **identity?** an equation that is true for <u>ALL</u> values of the variable.

What's a **conditional equation?** an equation that is only true for a certain number of values.

Fundamental Trigonometric Identities:

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

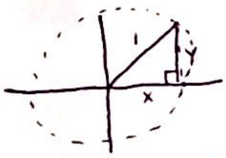
Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\frac{\sin\theta}{\cos\theta} = \frac{opp/hyp}{adj/hyp} = \frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj} = \frac{opp}{adj} = \tan$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1+\tan^2\theta = \sec^2\theta$ $\quad 1+\cot^2\theta = \csc^2\theta$



$x = \cos\theta$

$y = \sin\theta$

$x^2 + y^2 = 1$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$ $\quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$ $\quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$ $\quad \csc\left(\frac{\pi}{2} - u\right) = \sec u$

Even/Odd Identities

$\sin(-u) = -\sin u$

$\cos(-u) = \cos u$

$\tan(-u) = -\tan u$

$\csc(-u) = -\csc u$

$\sec(-u) = \sec u$

$\cot(-u) = -\cot u$

* <u>CO</u>function pairs will give you the same answer if using <u>Co</u>mplementary angles.

Example: $\sin 30^\circ = \cos 60^\circ$

$\frac{1}{2} = \frac{1}{2}$

Examples:

1. Use the given values and the fundamental identities to find all six trigonometric function values.

$\sec x = -\dfrac{3}{2}$ $\tan x > 0$ Quadrant III

$$\boxed{\cos x = -\frac{2}{3}, \quad \sin x = \frac{\sqrt{5}}{-3}, \quad \csc = \frac{-3\sqrt{5}}{5}, \quad \tan x = \frac{+\sqrt{5}}{2}, \quad \cot x = \frac{+2\sqrt{5}}{5}}$$

$\sin^2 x + \left(-\frac{2}{3}\right)^2 = 1$

$\sin^2 x = 1 - \frac{4}{9}$

$\sin^2 x = \frac{5}{9}$

$\sin x = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{-3}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\frac{\sqrt{5}}{-3}}{\frac{-2}{3}} = \dfrac{+\sqrt{5}}{2}$

2. Factor each of the following expressions.

a. $\sec^2 x - 1$

$\boxed{(\sec x + 1)(\sec x - 1)}$

(diff. of □'s)

b. $4\tan^2 x + \tan x - 3$

$\boxed{(\tan x + 1)(4\tan x - 3)}$

(trinomial)

c. $\csc^2 x - \cot x - 3$

$= 1 + \cot^2 x - \cot x - 3$

$= \cot^2 x - \cot x - 2$

$= (\cot x - 2)(\cot x + 1)$

3. Use the fundamental identities to simplify the expressions.

a. $\sin x \cos^2 x - \sin x$

$= \sin x(\cos^2 x - 1)$

$= \sin x(\cancel{1} - \sin^2 x \cancel{-1})$

$= \boxed{-\sin^3 x}$

b. $\sin y + \cot y \cos y$

$= \sin y + \dfrac{\cos y}{\sin y} \cdot \cos y$

$= \sin y + \dfrac{\cos^2 y}{\sin y}$

$= \sin y + \dfrac{1 - \sin^2 y}{\sin y}$

$\dfrac{\sin^2 y}{\sin y} + \dfrac{1 - \sin^2 y}{\sin y}$

$\dfrac{1}{\sin y} = \boxed{\csc y}$

c. $\cot^2 x - \csc^2 x$

$= \cot^2 x - (1 + \cot^2 x)$

$= \cot^2 x - 1 - \cot^2 x$

$= \boxed{-1}$

d. $\cos^2\theta(\sec^2\theta - 1)$

$= \cos^2\theta(\tan^2\theta)$

$= \dfrac{\cancel{\cos^2\theta}}{1}\left(\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\right)$

$= \boxed{\sin^2\theta}$

e. $\sin\beta(\csc\beta - \sin\beta)$

$= \sin\beta\left(\dfrac{1}{\sin\beta} - \sin\beta\right)$

$= \dfrac{\sin\beta}{\sin\beta} - \sin^2\beta$

$= 1 - \sin^2\beta = \boxed{\cos^2\beta}$

f. $\sin\left(\dfrac{\pi}{2} - x\right)\csc x$

$= \cos x \csc x$

$= \cos x \cdot \dfrac{1}{\sin x}$

$= \dfrac{\cos x}{\sin x} = \boxed{\cot x}$

Day 2: Verifying Trigonometric Identities

In this section, we will study techniques for verifying (proving) trigonometric identities. Remember, identities are statements that are true for **all** values of a given variable, so our goal will simply be to show that one side of the identity can be rewritten to match the other side.

NOTE: **In this section, your WORK is the answer. Problems showing insufficient steps will not be given credit. You must show each step you use in the proof. Skipping steps is NOT a good idea!**

Guidelines

1. Work only with one side of the identity at a time. (Often the more complicated side first.)

2. Look for opportunities to factor, add fractions, FOIL, or split fractions, etc.

3. Look for places to use the identities....you can use them to make substitutions.

5. If nothing else seems to help, try converting everything to sines and cosines.

6. Try ***something***!! Just staring at the problem isn't going to make anything happen. ☺

Examples: Verify the following identities.

1. $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$= \dfrac{\tan^2\theta}{\sec^2\theta}$

$= \dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}}$

$= \dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{1}$

$= \sin^2\theta$

2. $(\tan^2 y + 1)(\cos^2 y - 1) = -\tan^2 y$

$= (\sec^2 y)(1 - \sin^2 y - 1)$

$= (\sec^2 y)(-\sin^2 y)$

$= \dfrac{1}{\cos^2 y}(-\sin^2 y)$

$= \dfrac{-\sin^2 y}{\cos^2 y}$

$= -\tan^2 y$

Jay 3:

3. $\dfrac{1}{1-\sin x}\cdot\dfrac{1+\sin x}{} + \dfrac{1}{1+\sin x}\cdot\dfrac{1-\sin x}{} = 2\sec^2 x$

LCD: $(1+\sin x)(1-\sin x)$
$= 1-\sin^2 x$

$= \dfrac{1+\sin x}{(1-\sin^2 x)} + \dfrac{1-\sin x}{(1-\sin^2 x)}$

$= \dfrac{1+\sin x+1-\sin x}{1-\sin^2 x}$

$= \dfrac{2}{\cos^2 x}$

$= 2\sec^2 x$

4. $\tan x + \cot x = \sec x \csc x$

$= \dfrac{\sin x \cdot \sin x}{\sin x \cdot \cos x} + \dfrac{\cos x \cdot \cos x}{\sin x \cdot \cos x}$

$= \dfrac{\sin^2 x}{\cos x \sin x} + \dfrac{\cos^2 x}{\cos x \sin x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$= \dfrac{1}{\cos x \sin x}$

$= \sec x \csc x$

5. $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}\dfrac{(1+\sin x)}{(1+\sin x)}$ * must be conjugate

$= \dfrac{(\cos x)(1+\sin x)}{1-\sin^2 x}$

$= \dfrac{\cos x(1+\sin x)}{\cos^2 x}$

$= \dfrac{1+\sin x}{\cos x}$

$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$= \sec x + \tan x$

6. $\dfrac{\cot^2\theta}{1+\csc\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

$= \dfrac{\csc^2\theta - 1}{1+\csc\theta}$

$= \dfrac{\cancel{(\csc\theta+1)}(\csc\theta-1)}{\cancel{1+\csc\theta}}$

$= \csc\theta - 1$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$

$= \dfrac{1-\sin\theta}{\sin\theta}$

7. $\tan^4\alpha = \tan^2\alpha\sec^2\alpha - \tan^2\alpha$

$= \tan^2\alpha(\sec^2\alpha - 1)$

$= \tan^2\alpha(\tan^2\alpha)$

$= \tan^4\alpha$

8. $\sin^3\beta\cos^4\beta = (\cos^4\beta - \cos^6\beta)\sin\beta$

$= \cos^4\beta(1-\cos^2\beta)\sin\beta$

$= \cos^4\beta(\sin^2\beta)\sin\beta$

$= \sin^3\beta\cos^4\beta$

Day 3: **Solving Trigonometric Equations**

Solving an equation means to find all values of the variable that make the equation true.
With trig equations make sure you pay attention to the instructions… have you been given an interval???

Examples: *Solve each equation for all values of the variable on the interval $[0, 2\pi)$* – radians (one rotation)

1. $2\sin x - 1 = 0$

$\sin x = 1/2$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

2. $\sin x + \sqrt{2} = -\sin x$

$2\sin x + \sqrt{2} = 0$

$\sin x = -\sqrt{2}/2$

ref $\angle = \frac{\pi}{4}$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

3. $3\tan^2 x - 1 = 0$

ref $\angle = \pi/6$

$\tan^2 x = 1/3$

$\tan x = \pm\frac{1}{\sqrt{3}} = \frac{\pm\sqrt{3}}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

4. $\cot x \cos^2 x = 2\cot x$

$\cot x \cos^2 x - 2\cot x = 0$

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$ $\quad$ $\cos^2 x - 2 = 0$

$\boxed{x = \pi/2, 3\pi/2}$ $\quad$ $\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$

$\varnothing$ bigger than 1

5. $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2$ $\quad$ $\sin x = 1$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6} \quad x = \pi/2}$

Examples: *Solve each equation for **all** values of the variable.* Every possible angle

6. $2\cos x - 1 = 0$

$\cos x = 1/2$

ref $\angle = \pi/3$

Q I IV all coterminals

$\boxed{\begin{aligned} x &= \pi/3 + 2\pi n \\ x &= 5\pi/3 + 2\pi n \end{aligned}}$

7. $\sin^2 x = 2\sin x$

$\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad$ $\sin x = 2$

$\varnothing$ bigger

$\boxed{\begin{aligned} x &= 0 + 2\pi n \\ &\pi + 2\pi n \end{aligned}}$

8. $2\sin^2 x + 3\cos x - 3 = 0$

$2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$-(2\cos^2 x - 3\cos x + 1) = 0$

$-(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = 1/2$ $\quad$ $\cos x = 1$

$x = \pi/3 + 2\pi n$ $\quad$ $x = 0 + 2\pi n$

$5\pi/3 + 2\pi n$

9. $3\sec^2 x - 2\tan^2 x - 4 = 0$

$3(1 + \tan^2 x) - 2\tan^2 x - 4 = 0$

$3 + 3\tan^2 x - 2\tan^2 x - 4 = 0$

$\tan^2 x - 1 = 0$

$(\tan x + 1)(\tan x - 1) = 0$

$\tan x = -1$ $\quad$ $\tan x = 1$

$x = \frac{3\pi}{4} + 2\pi n$ $\quad$ $x = \pi/4 + 2\pi n$

$\frac{7\pi}{4} + 2\pi n$ $\quad$ $5\pi/4 + 2\pi n$

Multiple Angle Equations: Watch for a # multiplied or divided with a variable.

Day 4:

Examples: *Solve the equations for all values of the variable on the interval [0, 2π).*

10. $2\cos 3x - 1 = 0$ Q I, IV ref = $\pi/3$

$\cos 3x = 1/2$

$\frac{3x}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$

3

$\boxed{x = \pi/9,\ 5\pi/9,\ 7\pi/9,\ 11\pi/9,\ 13\pi/9,\ 17\pi/9}$

Need to list 3 times as many angles

11. $3\tan\frac{x}{2} + 3 = 0$ Q II, IV ref = $\pi/4$

$\tan\left(\frac{x}{2}\right) = -1$ → use half as many angles.

$\cancel{2} \cdot \frac{x}{\cancel{2}} = \left[3\pi/4,\ \frac{7\pi}{4}\right] \cdot 2$

$x = \frac{6\pi}{4} = \boxed{\frac{3\pi}{2}}$

$\frac{7\pi}{2}$ isn't between $[0, 2\pi)$

Solve the equations for all values of the variable.

12. $\sin 2x - \frac{\sqrt{3}}{2} = 0$ Q I, II ref = $\pi/3$

$\sin 2x = \sqrt{3}/2$

$2x = \frac{\pi}{3} + 2\pi n$

$\boxed{x = \frac{\pi}{6} + \pi n}$

&

$2x = \frac{2\pi}{3} + 2\pi n$

$\boxed{x = \frac{\pi}{3} + \pi n}$

13. $\cot\frac{x}{2} - 1 = 0$ Q I, III ref = $\pi/4$

$\cot\frac{x}{2} = 1$

$\frac{x}{2} = \frac{\pi}{4} + 2\pi n$

$\boxed{x = \frac{\pi}{2} + 4\pi n}$

$\frac{x}{2} = \frac{5\pi}{4} + 2\pi n$

$\boxed{x = \frac{5\pi}{2} + 4\pi n}$

Day 4: Sum and Difference Identities

Sum and Difference Identities:

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

NOTE: The distributive property does NOT apply to trig functions... $\sin(u+v) \neq \sin u + \sin v$

DO NOT DO THIS!!!

no calc :)

Examples:

Find the **exact value** of the following.

1. $\cos 75^\circ$

$= \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

2. $\sin \frac{\pi}{12}$

$\left(\frac{\pi}{12} = 15^\circ \;/\; 45^\circ - 30^\circ = 15\right)$

$= \sin 15^\circ = \sin(45^\circ - 30^\circ)$

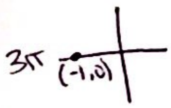
$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

3. Simplify $\tan(\theta + 3\pi)$ using the sum and difference formulas.

$$= \frac{\tan\theta + \tan 3\pi}{1 - \tan\theta \tan 3\pi} = \frac{\tan\theta + 0}{1 - \tan\theta(0)} = \frac{\tan\theta}{1} = \boxed{\tan\theta}$$



4. Solve: $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$ in the interval $[0, 2\pi)$.

$\sin x \cos\frac{\pi}{4} + \cancel{\sin\frac{\pi}{4}\cos x} + \sin x \cos\frac{\pi}{4} - \cancel{\sin\frac{\pi}{4}\cos x} = -1$

$2 \sin x \cos\frac{\pi}{4} = -1$

$2 \sin x \left(\sqrt{2}/2\right) = -1$

$\sqrt{2} \sin x = -1$

$\sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

ref ∠: $\frac{\pi}{4}$ Q ~~I, II~~ III IV

$\boxed{x = \cancel{\frac{\pi}{4}, \frac{3\pi}{4}}}$ $\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

(No interval so ALL angles)

5. Solve: $\cos\left(x+\frac{\pi}{6}\right)-\cos\left(x-\frac{\pi}{6}\right)=1$

$\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6} - \left(\cos x \cos \pi/6 + \sin x \sin\frac{\pi}{6}\right) = 1$

$\cancel{\cos x \cos\frac{\pi}{6}} - \sin x \sin\frac{\pi}{6} - \cancel{\cos x \cos\frac{\pi}{6}} - \sin x \sin\frac{\pi}{6} = 1$

$-2\sin x \sin\frac{\pi}{6} = 1$

$-2\sin x\left(\frac{1}{2}\right) = 1$

$-\sin x = 1$

$\sin x = -1$

$\boxed{x = \frac{3\pi}{2} + 2\pi n}$

6. Verify the identity: $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

$= \sin x\cos y + \sin y\cos x + \sin x\cos y - \sin y\cos x$

$= \sin x\cos y + \sin x\cos y$

$= 2\sin x\cos y$

no calc

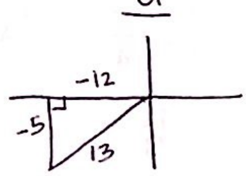
7. Find the exact value of $\cos(v-u)$ given that $\sin u=-\frac{5}{13}$ and $\cos v=-\frac{3}{5}$ and both u and v are in Quadrant III.

$\cos(v-u) = \cos v\cos u + \sin v\sin u$

$= \left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

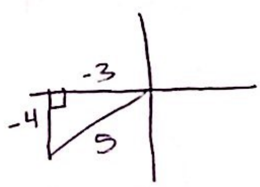
$= \frac{36}{65} + \frac{20}{65} = \boxed{\frac{56}{65}}$

u



$\cos u = \frac{-12}{13}$

v



$\sin v = \frac{-4}{5}$

Day 5: Multiple-Angle Formulas

Double-Angle Formulas:

$\sin 2u = 2\sin u\cos u$

$\cos 2u = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

* Can use any of these for substitution

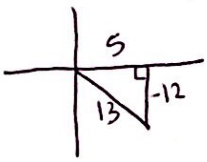
NOTE: The double angle identities provide a way to evaluate, for example, the sine of 480 if I happen to know the sine of 240. In other words, I can get the value for sine, cosine, or tangent of an angle that is double an angle I already have information about.

Examples:

1. Given $\cos\theta = \frac{5}{13}$, $\frac{3\pi}{2} < \theta < 2\pi$, find the values for $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$. Q IV

$\sin 2\theta = 2\sin\theta\cos\theta = 2(-12/13)(5/13) = \boxed{\frac{-120}{169}}$

$\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{5}{13}\right)^2 - 1 = \frac{50}{169} - 1$
$= \boxed{\frac{-119}{169}}$



$\sin\theta = \frac{-12}{13}$

$\tan\theta = \frac{-12}{5}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(-12/5)}{1-(-12/5)^2} = \dfrac{-24/5}{-119/25} = \boxed{\frac{120}{119}}$

2. Solve: $2\cos x + \sin 2x = 0$ (ALL since no interval)

$2\cos x + 2\sin x\cos x = 0$

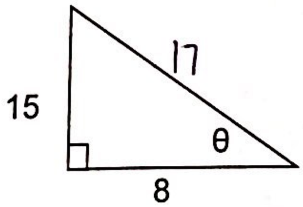
$\cos x(2 + 2\sin x) = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \frac{3\pi}{2} + 2\pi n$ same

$$\boxed{\begin{aligned} x &= \frac{\pi}{2} + 2\pi n \\ &\frac{3\pi}{2} + 2\pi n \end{aligned}}$$

3. Find the exact value of $\cos 2\theta$.



$15^2 + 8^2 = h^2$

$17 = h$

$\cos 2\theta = 1 - 2\sin^2\theta$
$= 1 - 2(15/17)^2$
$= 1 - 2\left(\frac{225}{289}\right)$
$= \boxed{\frac{-161}{289}}$

Half-Angle Formulas:

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

Examples:

→ $105°$ is in Q II, so answer is "+".

4. Find the **exact value** of $\sin 105°$ using the half-angle formulas.

$$\sin(105°) = \sin\left(\frac{210°}{2}\right) = +\sqrt{\frac{1-\cos 210°}{2}} = \sqrt{\frac{1-(-\sqrt{3}/2)}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

5. Find all solutions of $2-\sin^2 x = 2\cos^2\frac{x}{2}$ on the interval $[0, 2\pi)$.

$$0 = 2\cos^2\frac{x}{2} + \sin^2 x - 2$$

$$= 2\left(\sqrt{\frac{1+\cos x}{2}}\right)^2 + \sin^2 x - 2$$

$$= 2\left(\frac{1+\cos x}{2}\right) + \sin^2 x - 2$$

$$0 = 1 + \cos x + 1 - \cos^2 x - 2$$

$$0 = -\cos^2 x + \cos x$$

$$0 = \cos x(-\cos x + 1)$$

$\cos x = 0 \qquad \cos x = 1$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, 0}$$