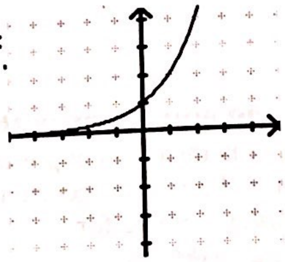
Unit 9
Exponential and Logarithmic Functions

Day 1
Exponential Functions and Graphs

The **exponential function with base a** is written as $f(x) = a^x$, where $a > 0$, $a \neq 1$, and x is any real number.

Exponential Function: $y = a^x$

Graph:



For exponential functions, the base is constant and the exponent contains a variable.

Be careful. Exponential functions: $y = 2^x, y = 3^x, y = \left(\frac{1}{2}\right)^x$, etc.

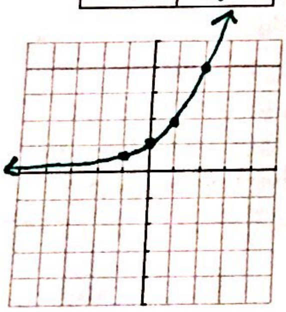
are different than power functions: $y = x^2,\ y = x^3,\ y = x^{\frac{1}{2}}$, etc.

Graphing Exponential Functions

Examples. Graph the following exponential functions. Find the domain, range, *y*-intercept and asymptote.

1. $y = 2^x$

x	y
−1	1/2
0	1
1	2
2	4



Domain: $(-\infty, \infty)$

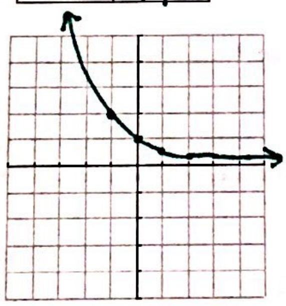
Range: $(0, \infty)$

y-intercept: $(0, 1)$

Horizontal asymptote: $y = 0$

2. $y = \left(\frac{1}{2}\right)^x$

x	y
−1	2
0	1
1	1/2
2	1/4



Domain: $(-\infty, \infty)$

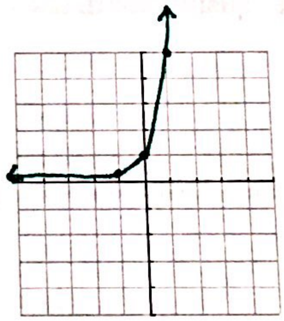
Range: $(0, \infty)$

y-intercept: $(0, 1)$

Horizontal asymptote: $y = 0$

3. $y = 5^x$

x	y
−1	1/5
0	1
1	5



Domain: $(-\infty, \infty)$

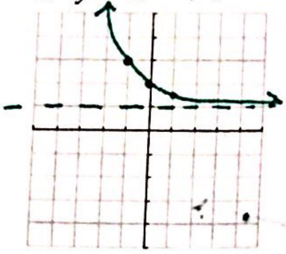
Range: $(0, \infty)$

y-intercept: $(0, 1)$

Horizontal asymptote: $y = 0$

Examples. Use transformations and the graphs in examples 1-3 to graph the following functions. Determine the y-intercept and the equation of the horizontal asymptote:

4. $y = 2^{-x} + 1$



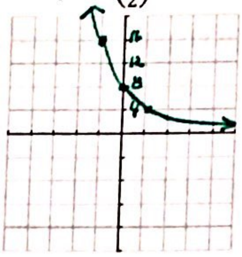
x	y
-1	3
0	2
1	3/2

y-intercept: (0, 2)

Horizontal asymptote: y = 1

Transformations: flip over y up 1

5. $y = \left(\frac{1}{2}\right)^{x-3}$



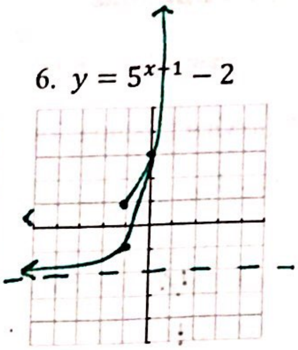
x	y
-1	16
0	8
1	4

y-intercept: (0, 8)

Horizontal asymptote: y = 0

Transformations: right 3

6. $y = 5^{x+1} - 2$



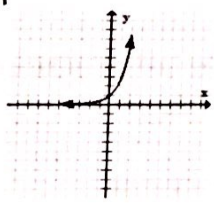
x	y
-1	-1
0	3
1	23

y-intercept: (0, 3)

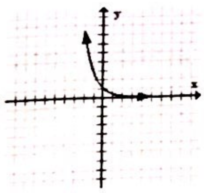
Horizontal asymptote: y = -2

Transformations: left 1, down 2

Exponential Growth Model
(increasing function)



Exponential Decay Model
(decreasing function)



Look at the graphs in examples 1-6. Which of the functions are exponential growth models?

1, 3, 6 → growth   2, 4, 5 → decay

What kind of equations will produce exponential decay models?

fractional bases, negative exponents

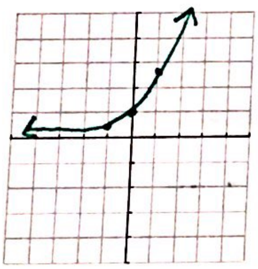
The Natural Base *e*

In scientific applications, statistics and calculus, the most commonly used base for exponential functions is base *e*. It is known as the **natural base**. *e* is an irrational number, like π or $\sqrt{2}$

$\boxed{e \approx 2.718}$

Examples. Graph $y = e^x$. Then use transformations to graph the other functions. Find the y-intercept and horizontal asymptote.

7. $y = e^x$

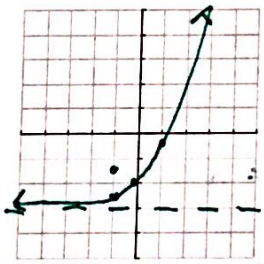


x	y
-1	.37
0	1
1	2.7

y-intercept: (0, 1)

horizontal asymptote: y = 0

8. $y = e^x - 3$



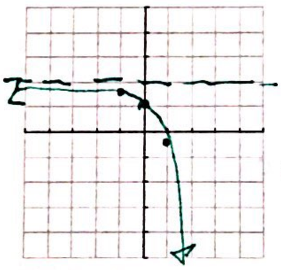
x	y
-1	-2.6
0	-2
1	-.28

y-intercept: (0, -2)

horizontal asymptote: y = -3

down 3

9. $y = -e^x + 2$



x	y
-1	1.63
0	1
1	-.72

y-intercept: (0, 1)

horizontal asymptote: y = 2

flip over x up 2

Day 2

Logarithmic Functions

Since basic exponential functions are one-to-one, they must have inverses.

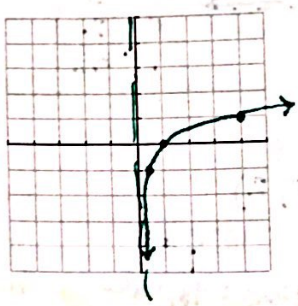
Graph the following function. Then use the graph to graph the inverse function.

$f(x) = 4^x$

x	y
-1	$1/4$
0	1
1	4

$f^{-1}(x)$

x	y
$1/4$	-1
1	0
4	1



We will graph these on day 3.

The inverse graph is an example of a **logarithmic graph**. The inverse of an exponential function with base a is known as a **logarithmic function with base a** ($f(x) = \log_a x$).

$$y = \log_a x \leftrightarrow x = a^y$$

Basic Properties of Logarithms

$\log_a x = y$

The definition: $y = \log_a x \leftrightarrow x = a^y$ allows you to convert from logarithmic to exponential form and from exponential to logarithmic form. You need to be able to do this without notes!!

REMEMBER: THE LOG IS EQUAL TO THE EXPONENT

Examples. Write each equation in exponential form:

1. $\log_3 x = 2$ $x = 3^2$

   $3^2 = x$

2. $3 = \log_4 64$

   $4^3 = 64$

3. $\log_5 1 = 0$

   $5^0 = 1$

Examples. Write each equation in logarithmic form:

4. $6^0 = 1$

   $\log_6 1 = 0$

5. $4^x = 16$

   $\log_4 16 = x$

6. $3^{-4} = \frac{1}{81}$

   $\log_3 \frac{1}{81} = -4$

Examples. Convert from logarithmic form to exponential form and then determine x:

7. $\log_3 27 = x$

   $3^x = 27$

   $x = 3$

8. $\log_5 1 = x$

   $5^x = 1$

   $x = 0$

9. $\log_4 2 = x$

   $4^x = 2$

   $x = \frac{1}{2}$

Examples. Evaluate the logarithm without a calculator:

10. $\log_8 64 = 2$ $8^{?} = 64$

11. $\log_4\left(\frac{1}{4}\right) = -1$ $4^{?} = 1/4$

12. $\log_2 8 = 3$ $2^{?} = 8$

13. $\log_9 3 = 1/2$ $9^{?} = 3$

Common Bases
Two bases are used so often in applications that they are included in every scientific and graphing calculator. They are the common base (10) and the natural base (e).

Common Log: $\log_{10} x = \log x$	**Natural Log:** $\log_e x = \ln x$

Examples. Use a calculator to evaluate:

14. $\log 5$ .69897

15. $\ln e^5$ 5

16. $2 \ln 4.6$ 3.05211

Assignment 9.2

Day 3

Graphing Logarithmic Functions

$$y = \log_a x \leftrightarrow x = a^y$$

> **Common characteristics of graphs of logarithmic functions:**
> **Domain is limited**
> **Vertical asymptote**
> **(1, 0) is a point on the graph (unless it's shifted or reflected)**

Examples.

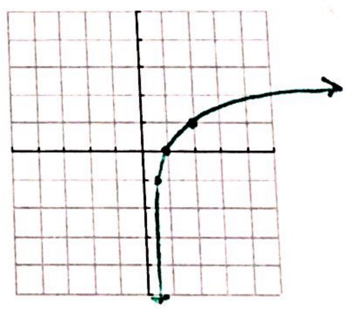
1. Find points for $f(x) = 2^x$. Then switch the x and y values in order to graph $g(x) = \log_2 x$. Write an equation for the vertical asymptote and list the domain.

$f(x) = 2^x$

x	y
−1	1/2
0	1
1	2

$g(x) = \log_2 x$

x	y
1/2	−1
1	0
2	1



Vertical asymptote: $x = 0$

Domain: $(0, \infty)$

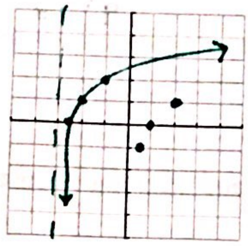
Range: $(-\infty, \infty)$

x-int: $(1, 0)$

Examples. Use the graph in example 1 and transformations to graph the following functions. List the domain and vertical asymptote.

up 1 left 3

2. $f(x) = 1 + \log_2(x+3)$



x	y
½	−1
1	0
2	1

Vertical asymptote: $x = -3$

Domain: $(-3, \infty)$

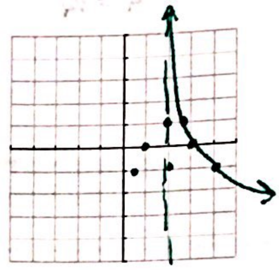
Range: $(-\infty, \infty)$

x-int: $(-5/2, 0)$

reflect over x-axis

right 2

3. $y = -\log_2(x-2)$



x	y
½	−1
1	0
2	1

Vertical asymptote: $x = 2$

Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

x-int: $(3, 0)$

***Example.* Graphing $f(x) = \ln x$**

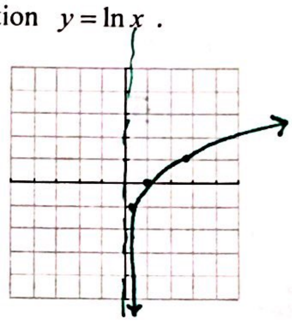
4. Use points from $y = e^x$ to graph the inverse function $y = \ln x$.

$y = e^x$

x	y
−1	.37
0	1
1	2.7

$y = \ln x$

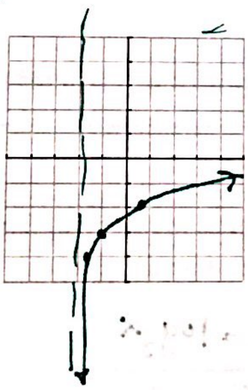
x	y
.37	−1
1	0
2.7	1



Examples. Use the graph in example 4 and transformations to graph the following functions. List the domain and vertical asymptote.

5. $f(x) = \ln(x+2) - 3$ down 3

left 2



Domain: $(-2, \infty)$

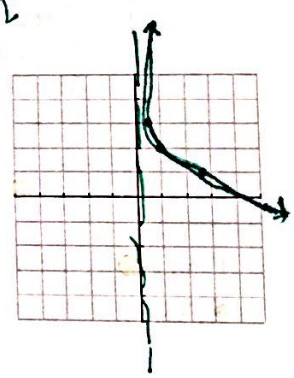
Vertical asymptote: $x = -2$

Range: $(-\infty, \infty)$

reflect over thru x-axis

up 2

6. $f(x) = -\ln(x) + 2$



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

Range: $(-\infty, \infty)$

main

te:

Domain of Logarithmic Functions

The domain of $y = \log_a x$ is $x > 0$. The argument of a log function must be positive.

- the inside must be positive -

Examples. Find the domain for each of the following logarithmic functions:

7. $f(x) = \ln(x + 3)$

$x + 3 > 0$
$-3 \quad -3$
$\boxed{x > -3}$
or $(-3, \infty)$

8. $y = \ln|x|$

$(-\infty, \infty)$

9. $g(x) = \ln(2 - 3x)$

$2 - 3x > 0$
$-2 \quad\quad -2$
$\frac{\cancel{-3}x}{\cancel{-3}} > \frac{-2}{-3}$
$x < 2/3$
$(-\infty, 2/3)$

Assignment 9.3

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Day 4

Unit 9 Test ***All late/absent assignments due for Unit 9***

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